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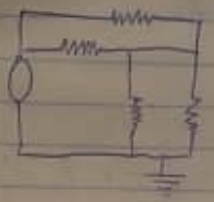
17/ENG02/006
COMPUTER ENGINEERING
CIRCUIT THEORY ASSIGNMENT

① Find the Voltage at node 1, 2, 3 ... the circuit below



Node 1 KCL

$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{3}$

Multiply through by 6

$6 \times 10 = 3(V_1 - V_3) + 2(V_1 - V_2)$

$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$

$60 = 5V_1 - 3V_3 - 2V_2$ — (i)

At Node 2 KCL

$I_2 = I_3 + I_4$

$I_4 = I_2 - I_1$

$I_4 = \frac{V_1 - V_2}{3} - \frac{V_3 - 0}{4}$

$I_4 = -1.2A$

Multiply through by 7

$7 \times 8 = 4(V_1 - V_2) - 3(V_3 - 0)$

$7 \times 8 = 4V_1 - 7V_2$ — (ii)

At Node 3 KCL

$I_4 + I_1 = I_5$

$I_4 = I_5 - I_1$

$I_4 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$

$3I_4 = V_3 - 3(V_1 - V_3)$

$3I_4 = -3V_1 + 4V_3$ — (iii)

$5V_1 - 2V_2 - 3V_3 = 60$ —— ①

$4V_1 - 7V_2 + 0V_3 = 768$ —— ②

$-3V_1 + 0V_2 + 4V_3 = 384$

Matrix form

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$ $V_2 = \frac{\Delta_2}{\Delta}$ $V_3 = \frac{\Delta_3}{\Delta}$

$$\Delta = \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{vmatrix}$$

$= 5(-28-0) + 2(16+0) - 3(0-21)$

$= -140 + 32 + 63$

$= -45$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix}$$

$= 60(-28-0) - 768(-8-0) - 384(0-21)$

$= -1680 + 6144 + 8064$

$= -3600$

$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

$$V_2 = \Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$$

$= 5((768 \times 4) - 0) - 4((60 \times 4) - (384 - 3)) - 3(0 - (768 \times -3))$

$= 2880$

$V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

$$V_3 = \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$$

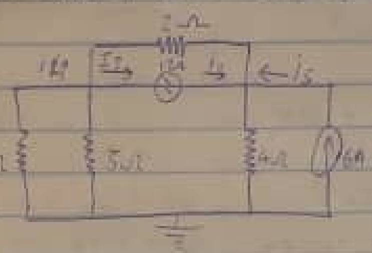
$= 5((-7\times384) - 0) - 4((-2\times384) - 0) - 3((-2\times768) - (-7\times60))$

$= -7020$

$V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$

$\therefore V_1 = 80V,\ V_2 = -64V,\ V_3 = 156V$

(2)



Node 1 KCL

$i_1 = i_2 + 12 + i_4$

$\frac{V_3 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_2}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2$ —— (i)

Node 2

$i_2 + i_3 + i_5 = i$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 + 48 = 2V_2$

$144 = -4V_1 + 6V_2$ —— (ii)

Using Elimination method

$120 = -8V_1 + 5V_2$ —— (i) ×4

$144 = -4V_1 + 6V_2$ —— (ii) ×8

$-480 = 32V_1 + 20V_2 + 20V_2$ —— (iii)

$-1152 = 32V_1 - 48V_2$ —— (iv)

Substitute eq (3) in (2)

$-672 = 0 - 28V_2$

$-672 = -28V_2$

$V_2 = \frac{-672}{-28}$

$V_2 = 24V$

Sub V_2 into eq (1)

$144 = -4V_1 + 6V_2$

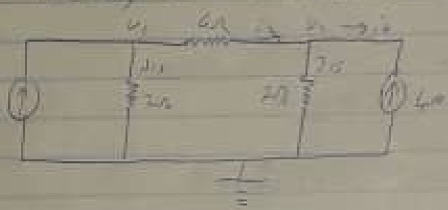
$V_1 = \frac{144 - 6(24)}{-4}$

$V_1 = \frac{144 - 144}{-4} = 0V$

$V_1 = 0V$; $V_2 = 24V$

$i_1 = 0A$, $i_2 = 0A$, $i_3 = 6A$, $i_4 = -12A$

ii)



Node 1

$i_1 = i_2 + i_3$

$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{3}$

$6 = V_1 - V_2 + 2V_1$

$6 = 3V_1 - V_2$ — (1)

Node 2

$i_2 = i_4 + i_3$

$\frac{V_1 - V_2}{6} = \frac{4 + V_2}{7}$

$7(V_1 - V_2) = 6(4 + V_2)$

$7V_1 - 7V_2 = 24 + 6V_2$

$168 = 7V_1 - 13V_2 - 78$

From eqn 2

$V_2 = 4V_1 - 6$

Sub $V_2 = 4V_1 - 6$ into eqn ②

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45V_1$

$V_1 = \frac{90}{-45}$

$V_1 = -2V$

Sub $V_1 = -2$ into eqn 1

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14V$

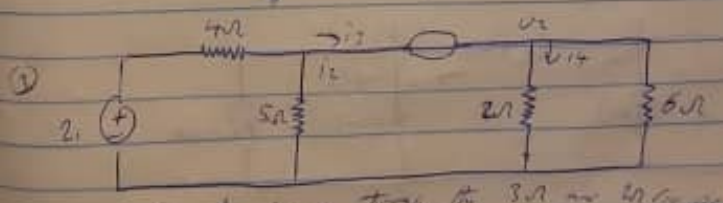
$V_1 = -2V$, $V_2 = -14V$

Current through the 6Ω resistor

$i_{6\Omega} = \frac{V_1 - V_2}{6} = \frac{-2 + 14}{6} = 2A$

$i_{3\Omega} = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_{7\Omega} = \frac{V_2}{7} = \frac{-14}{7} = -2A$



For the current through the 5Ω and 2Ω resistor

Use KCL at Node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 2_1}{4} = \frac{V_1}{3} + \frac{V_1}{6} + \frac{V_2}{2}$