

$$\begin{array}{r} x^3 - x^2 \\ \underline{2x^2 + x + 1} \\ 2x^2 - 2x \end{array}$$

$$\begin{array}{r} 3x + 1 \\ \underline{- 3x - 3} \\ 4 \end{array}$$

which can now be written as

$$\int (x^3 + 2x^2 + 2x + 1) dx + \int \frac{4}{x-1} dx$$

$$\Rightarrow \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{1}$$

$$\Rightarrow \frac{x^3}{3} + x^2 + 3x + 4$$

$$\Rightarrow \int (x^2 + 2x + 3) dx + \int \frac{4}{x-1} dx$$

$$\Rightarrow \frac{x^3}{3} + x^2 + 3x + 4 \ln|x-1| + C$$

$$\begin{array}{r}
 x^2 + \cancel{2x} + 2x + 2 \\
 x-1 \overline{) x^3 + x^2 + x + 1} \\
 \underline{x^3 - x^2} \\
 2x^2 + x + 1 \\
 \underline{2x^2 - 2x} \\
 3x + 1 \\
 \underline{3x - 3} \\
 4
 \end{array}$$

which can now be written as

$$\int \frac{(x^3 + 2x + 1) dx + \int 4}{x-1} dx$$

$$\Rightarrow \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{1}$$

$$\Rightarrow \frac{x^3}{3} + x^2 + 3x + 4$$

$$\rightarrow \int (x^2 + 2x + 3) dx + \int \frac{4}{x-1} dx$$

$$\Rightarrow \frac{x^3}{3} + x^2 + 3x + 4 \ln|x-1| + C$$

$$2x - B + C = 4$$

$$-1 \times 2B - 3C = -3$$

$$-2B + 2C = 8$$

$$-2B + 3C = 3$$

$$-C = 5$$

$$C = -5$$

$$-B + C = 4$$

$$-B - 5 = 4$$

$$-B = 4 + 5$$

$$-B = 9$$

$$B = -9$$

$$A + B = 1$$

$$A = 1 - B$$

$$A = 1 - (-9)$$

$$A = 10$$

$$\int \frac{(x^2 + 1)}{(x-3)(x-2)^2} dx = \int \frac{10}{(x-3)} dx + \int \frac{-9}{(x-2)} dx + \int \frac{-5}{(x-2)^2} dx$$

$$= 10 \ln(x-3) - 9 \ln(x-2) - \frac{5}{x-2} + C$$

$$= 10 \ln(x-3) - 9 \ln(x-2) - \frac{5}{x-2} + C$$

$$B = \frac{5}{3}$$

$$B = -\frac{5}{3}$$

$$A + B - C = 0$$

$$A + \left(-\frac{5}{3}\right) + \frac{3}{2} = 0$$

$$A - \frac{1}{6} = 0$$

$$A = \frac{1}{6}$$

$$\int \frac{\frac{1}{6} dx + \left(\frac{5}{3}\right) dx + \frac{3}{2} dx}{x-1} + \frac{3}{2} dx$$

$$\frac{1}{6} \ln(x-1) - \frac{5}{3} \ln(x-2) + \frac{3}{2} \ln(x-3) + C$$

$$\textcircled{2} \int \frac{(x^2+x+1) dx}{(x+2)(x^2+1)} \quad \int \frac{(x^2+x+1) dx}{(x^2+1)(x+2)}$$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{Ax^2+Bx+C}{(x+2)} + \frac{D}{(x^2+1)} = \frac{(x^2+x+1) dx}{(x+2)(x^2+1)}$$

$$(Ax^2+Bx+C)(x^2+1) + D(x)$$

$$\frac{Ax^2+Bx+C}{(x^2+2)} + \frac{D}{(x+2)} = \frac{(x^2+x+1)}{(x^2+1)(x+2)}$$

$$\frac{(Ax^2+Bx+C)(x+2) + D(x^2+2)}{(x^2+2)(x+2)} = \frac{(x^2+x+1)}{(x^2+1)(x+2)}$$

$$C = 1 - \frac{2}{3}$$

$$C = \frac{1}{3}$$

$$B = -D$$

$$B = -\left(\frac{1}{3}\right)$$

$$B = -\frac{1}{3}$$

$$A = 0, B = -\frac{1}{3}, C = \frac{1}{3}, D = \frac{1}{3}$$

$$\int \frac{0 \, dx}{(x^2+1)} + \frac{\left(-\frac{1}{3}\right) dx}{x^2+1} + \frac{\left(\frac{1}{3}\right) dx}{x^2+1} + \frac{\frac{1}{3} dx}{x+2}$$

$$\Rightarrow -\frac{1}{3} \ln(x^2+1) + \frac{1}{3} \ln(x^2+1) - \frac{1}{3} \ln(x+2)$$

$$\textcircled{3} \frac{(x^2+1) \, dx}{(x-3)(x-2)^2}$$

$$\left(\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \right)$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x-2)^2 + B(x-2)(x-3) + C(x-3)}{(x-3)(x-2)^2}$$

$$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$$

Multiply B. $(x^2+1)(x+2)$

$$(Ax^2+Bx+C)(x+2) + D(x^2+1) = (x^2+x+1)$$

$$Ax^3 + 2Ax^2 + Bx^2 + 2Bx + Cx + 2C + Dx^2 + D = (x^2+x+1)$$

$$(A)x^3 + (2A+B) x^2 + (2B+C)x + (2C+D) = (x^2+x+1)$$

$$\text{Eq (1)} \quad A = 0 \quad \text{--- (1)}$$

$$2A + B + D = 1 \quad \text{--- (2)}$$

$$2B + C = 1 \quad \text{--- (3)}$$

$$2C + D = 1 \quad \text{--- (4)}$$

Put $A = 0$ in (2)

$$2(0) + B + D = 1$$

$$B + D = 1$$

$$B = -D$$

Put $B = -D$ in (3)

$$2(-D) + C = 1$$

$$-2D + C = 1 \quad \text{--- (5)}$$

$$C - 2D = 1 \quad \text{--- (6)}$$

By Elimination method

$$1 \times 2C + D = 1$$

$$2 \times C - 2D = 1$$

$$\begin{array}{r} 2C + D = 1 \\ - \end{array}$$

$$2C - 4D = 2$$

$$-3D = -1$$

$$D = \frac{-1}{-3}$$

$$D = \frac{1}{3}$$

$$C - 2D = 1$$

$$C - 2\left(\frac{1}{3}\right) = 1$$

$$(A+B+C)x^2 + (-5A-4B-3C)x + (6A+3B+2C) = 3x-1$$

$$A+B+C = 0$$

$$-5A-4B-3C = 3$$

$$6A+3B+2C = -1$$

$$A = -B-C$$

$$5(-B-C) - 4B - 3C = 3$$

$$-5B - 5C - 4B - 3C = 3$$

$$-9B - 8C = 3 \quad \text{--- (1)}$$

$$5(-B-C) - 4B - 3C = 3$$

$$-5B - 5C - 4B - 3C = 3$$

$$-9B - 8C = 3$$

$$6(-B-C) + 3B + 2C = -1$$

$$-6B - 6C + 3B + 2C = -1$$

$$-3B - 4C = -1 \quad \text{--- (2)}$$

~~By Elimination method~~

$$-3x - 9B - 8C = 3$$

$$-9x - 3B + 4C = -1$$

$$\Rightarrow \frac{-27B + 24C = -9}{-27B + 36C = 9}$$

$$-12C = -18$$

$$-5A = -1$$

By Elimination method

$$-3x - 9B - 8C = 3$$

$$-9x - 3B - 4C = -1$$

$$\Rightarrow \frac{-27B + 24C = -9}{-27B + 36C = 9}$$

$$-12C = -18$$

$$C = \frac{-18}{-12}$$

$$C = \frac{3}{2}$$

In eqn(2)

$$-3B - 4C = -1$$

$$-3B - 4\left(\frac{3}{2}\right) = -1$$

$$-3B - 6 = -1$$

$$-3B = 5$$

$$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$$

$$x^2+1 = (A+B)x^2 + (-4A-5B+C)x + 4A-6B-3C$$

$$x^2+1 = (A+B)x^2 + (-4A-5B+C)x + 4A-6B-3C$$

$$-4A-5B+C = 0 \quad (2)$$

$$4A+6B-3C = 1 \quad (3)$$

$$A = 1-B-C \quad (4)$$

$$-4(1-B) - 5B + C = 0$$

$$-4 + 4B - 5B + C = 0$$

$$-4 - B + C = 0$$

$$-4B + C = 4$$

$$-B + C = 4$$

$$-B + C = 4$$

$$C = 4 + B$$

$$4(1-B) - 6B - 3C = 1$$

$$4 - 4B - 6B - 3C = 1$$

$$4 - 10B - 3C = 1 \quad 4 + 2B - 3C = 1$$

$$-10B - 3C = -3 \quad 2B - 3C = -3$$

$$\div 10 \times -4B + C = 4$$

$$-10 \times -B + C = 4$$

$$-4 \times -10B - 3C = -3$$

$$-1 \times -10B - 3C = -3$$

$$+10B - 10C = -40$$

$$\rightarrow 40B - 10C = -40$$

$$-10B + 3C = 3$$

$$-40B + 12C = 12$$

$$-13C =$$

$$-22C = -52$$

$$-10B - 3(4 + B) = -3$$

$$-10B - 12 - 3B = -3$$

$$-10B - 3B = -3 + 12$$

$$-13B = 9$$