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18/ENG02/084

1 Vector Space: A vector space over a real fixed F is a set that is closed under finite vector addition and scalar multiplication.

2 $\alpha P + \beta Q + \gamma R = (a,b,c)$

$$\alpha\begin{bmatrix}1\\1\\1\end{bmatrix} + \beta\begin{bmatrix}1\\2\\3\end{bmatrix} + \begin{bmatrix}1\\5\\8\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$$

$$\alpha + \beta + \gamma = a \quad \text{—①}$$
$$\alpha + 2\beta + 5\gamma = b \quad \text{—②}$$
$$\alpha + 3\beta + 8\gamma = c \quad \text{—③}$$

From eqn (1)

$$\alpha = a - \beta - \gamma \cdots ④$$

put eqn ④ in eqn ② and eqn ③

In eqn ②

$$a - \beta - \gamma + 2\beta + 5\gamma = b$$
$$a + \beta + 4\gamma = b$$
$$\beta + 4\gamma = b - a \cdots ⑤$$

In eqn ③

$$a - \beta - \gamma + 3\beta + 8\gamma = c$$
$$a + 2\beta + 7\gamma = c$$
$$2\beta + 7\gamma = c - a \cdots ⑥$$

Combine eqn ⑤ & ⑥

$$2\times\ \beta + 4\gamma = b - a$$
$$1\times\ 2\beta + 7\gamma = c - a$$

$$2\beta + 8\gamma = 2b - 2a$$
$$-\ \ 2\beta + 7\gamma = c - a$$
$$\gamma = (2b - 2a) - (c - a)$$

$\gamma = 2b - 2a - c + a$

$\gamma = 2b - a - c$

$\gamma = -a + 2b - c$

From eqn ③

$\beta + 4\gamma = b - a$

$\beta + 4(-a + 2b - c) = b - a$

$\beta + (-4 + 8b - 4c) = b - a$

$\beta = (b - a) - (-4a + 8b - 4c)$

$= b - a + 4a - 8b + 4c$

$= b + 3a - 8b + 4c$

$= 3a - 7b + 4c$

From eqn 4)

$\alpha = a - \beta - \gamma$

$\alpha = \frac{a}{1} - \left(\frac{3a - 7b + 4c}{1}\right) - \left(\frac{-a + 2b - c}{1}\right)$

$= \frac{a - 3a + 7b - 4c + a - 2b + c}{1}$

2 $P = (1, 2, 3)$, $Q = (3, 2, 1)$, $R = (0, 0, 1)$

$\alpha P + \beta Q + \gamma R = (a, b, c)$

$$\alpha \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \beta \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \gamma \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$\alpha + 3\beta = a \quad \cdots ①$

$2\alpha + 2\beta = b \quad \cdots ②$

$3\alpha + \beta + \gamma = c \quad \cdots ③$

From eqn ①

$\alpha = a - 3\beta \quad \cdots ④$

put eqn ④ in ② and ③

In eqn ②

$2(a-3\beta)+2\beta=b$

$2a-6\beta+2\beta=b$

$-4\beta=b-2a \quad \cdots (5)$

In Eqn (3)

$3(a-3\beta)+\beta+\gamma=c$

$3a-6\beta+\beta+\gamma=c$

$3a-5\beta+\gamma=c$

$-5\beta+\gamma=c-3a \quad \cdots (6)$

Combine eqn (5) & (6)

$5\times -4\beta=b-2a$

$4\times -5\beta+\gamma=c-3a$

$-20\beta=(b-2a)5$

$-20\beta+4\gamma=4(c-3a)$

$-20\beta=5b-10a$

$-\quad -20\beta+4\gamma=4c-12a$

$4\gamma=(5b-10)-(4c-12a)$

$4\gamma=(5b-10a)-(4c-12a)$

$=5b-10a-4c+12a$

$4\gamma=5b-4c+2a$

$\gamma=\frac{5b-4c+2a}{4}$

From eqn (3)

$-5\beta+\gamma=c-3a$

$-5\beta+\frac{5b-4c+2a}{4}=c-3a$

$-5\beta=c-3a-\frac{5b-4c+2a}{4}$

$$-5\beta = c - 3a - \frac{5b - 4c + 2a}{4}$$

$$= \frac{4(c-3a) - 5b - 4c + 2a}{4}$$

$$= \frac{4c - 12a - 5b - 4c + 2a}{4}$$

$$= \frac{4c - 4c - 12a + 2a - 5b}{4}$$

$$-5\beta = \frac{-10a - 5b}{4}$$

$$\beta = \frac{1}{-5}\left(\frac{-10a - 5b}{4}\right)$$

$$\alpha = a - 3\beta$$

$$= a - 3\left(\frac{-10a - 5b}{-20}\right)$$

$$\alpha = a - 3\left(\frac{-10a - 5b}{-20}\right)$$