AKUMA SUNNY. U.
17/ENG04/009
ELECT/ELECT
ENG 322 ASSIGNMENT

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1) ~~Fo~~ At node 1, KCL:

$$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{3}$$

$$60 = 3(V_1 - V_3) + 2(V_1 - V_2)$$

$$60 = 3V_1 - 3V_2 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \quad \ldots (i)$$

At node 2, KCL:

$$i_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$$

$$768 = 4(V_1 - V_2) - 3(V_2 - 0)$$

$$768 = 4V_1 - 4V_2 - 3V_2$$

$$768 = 4V_1 - 7V_2 \quad \ldots (ii)$$

At Node 3, KCL

$$64 + i_1 = i_5$$

$$64 = i_5 - i_1$$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_2}{2}$$

$$384 = V_3 - 3(V_1 - V_3)$$

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$$384 = -3V_1 + 4V_3 \quad \ldots \text{(iii)}$$

Using crammer's rule

$$5V_1 - 2V_2 - 3V_3 = 60 \quad \ldots \text{(i)}$$

$$6V_1 - 7V_2 = 768 \quad \ldots \text{(ii)}$$

$$-3V_1 + 4V_3 = 384 \quad \ldots \text{(iii)}$$

In Matrix representation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$$

$$\begin{array}{cccc} & + & - & + \\ \text{where } \Delta = & \multicolumn{3}{c}{\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix}} \end{array}$$

$$= 5(-28-0) + 2(16+0) - 3(0-21)$$

$$= -140 + 32 + 63$$

$$= -45$$

$$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -4 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60(-28-0) - 768(-8-0) - 384(0-21)$$

$$= -1680 + 6144 - 8064$$

$$= -3600$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$$

for V_2 : $\Delta_2 = \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5((768\times4)-0)-4((60\times4)-(380\cdot-3))-3(0-(768\cdot-3))$

$= 2880$

$\therefore V_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

for V_3: $\begin{vmatrix} 5 & -2 & 60 \\ 4 & -1 & 768 \\ -3 & 0 & 384 \end{vmatrix}$

$= 5((-1\times384)-0)-4((-2\times384)-0)-3((-2\times768)-(-1\times60))$

$= -7020$

$\therefore V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$

Hence $V_1 = 80V$, $V_2 = 64V$, $V_3 = 156V$

2i At node 1, KCL

$I_1 = i_2 + i_3 + i_4$

$$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - 4}{5}$$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

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$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2 \quad \ldots \text{(i)}$

At node 2

$i_2 + i_3 + i_5 = i_6$

$$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$$

$96 + 4(V_1 - V_2) + 48 = 2(V_2)$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2 \quad \ldots \text{(ii)}$

Using Elimination method

$120 = -8V_1 + 5V_2 \quad \ldots \text{(i)} \qquad \times -4$

$144 = -4V_1 + 6V_2 \quad \ldots \text{(ii)} \qquad \times -8$

$-480 = 32V_1 - 20V_2 \quad \ldots \text{(iii)}$

$-1152 = 32V_1 - 48V_2 \quad \ldots \text{(iv)}$

Substract eqn (iii) from (iv)

$-672 = 0 - 28V_2$

$$V_2 = \frac{-672}{-28}$$

$V_2 = 24\,V$

Subs $V_2 = 24$ in eqn (ii)

$144 = -4V_1 + 6V_2$

$$V_1 = \frac{144 - 6V_2}{-4}$$

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$$V_1 = \frac{144 - 6V_2}{-4}$$

$V_1 = 0$

$\therefore V_1 = 0V$, $V_2 = 24V$

$i_1 = 0A$, $i_2 = 0A$, $i_3 = 6A$, $i_4 = -12A$

ii) At node 1

$i_1 = i_2 + i_3$

$$1 = \frac{V_1 - V_2}{6} + \frac{V_1}{2}$$

$6 = V_1 - V_2 + 3V_1$

$6 = 4V_1 - V_2$ --- (i)

At node 2

$i_2 = i_4 + i_5$

$$\frac{V_1 - V_2}{6} = 4 + \frac{V_2}{7}$$

$7(V_1 - V_2) = 168 + 6V_2$

$168 = 7V_1 - 13V_2$ --- (ii)

from eqn (i), $V_2 = 4V_1 - 6$

subs $V_2 = 4V_1 - 6$ in eqn (ii)

$168 = 7V_1 - 13(4V_1 - 6)$

$168 = 7V_1 - 52V_1 + 78$

$90 = -45 V_1$

$V_1 = 90 / -45$

$V_1 = -2V$

Subs. $V_1 = -2$ in equ(i)

$6 = 4(-2) - V_2$

$6 = -8 - V_2$

$V_2 = -8 - 6$

$V_2 = -14 V$

$\therefore V_1 = -2V,\ V_2 = -14V$

Current through the resistors,

$i_2 = \frac{V_1 - V_2}{6} = \frac{-2+14}{6} = 2A$

$i_3 = \frac{V_1}{2} = \frac{-2}{2} = -1A$

$i_5 = \frac{V_2}{7} = \frac{-14}{7} = -2A$

3) Using KCL at Node 1

$i_1 + i_2 + i_3 + i_4 = 0$

$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$

$7V_1 + 8V_2 - 63 = 0 \quad \ldots \text{(i)}$

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Using KVL for loop 2

$-V_1 - 9 + V_2 = 0$

$-V_1 + V_2 = 9$ .... (ii)

$7V_1 + 8V_2 = 63$ ... (i)

$-V_1 + V_2 = 9$ ... (ii)

in (ii) let $V_2 = 9 + V_1$

sub $V_2 = 9 + V_1$ in eqn (i)

$7V_1 + 8(9 + V_1) = 63$

$7V_1 + 72 + 8V_1 = 63$

$15V_1 = -9$

$V_1 = -0.6V$

Sub $V_1 = -0.6$ in eqn (ii)

$-(-0.6) + V_2 = 9$

$0.6 + V_2 = 9$

$V_2 = 8.4V$

$\therefore V_1 = -0.6V$ and $V_2 = 8.4V$

Current through the 3Ω resistor,

$i2 = \frac{V_1}{3} = \frac{-0.6}{3} = -0.2A$

Current through the 2Ω resistor

$i2 = \frac{V_2}{4} = \frac{8.4}{4} = 4.2A$

4) let assume that $V_1 - V_2 = 6V \Rightarrow i_2$

At node 1, using KCL

$6mA = i_1 + i_2$

$6mA = \frac{V_1 - 0}{6} + (V_1 - V_2)$

$36 = V_1 + 6(V_1 - V_2)$

$36 = V_1 + 6V_1 - 6V_2$

$36 = 7V_1 - 6V_2 \quad \text{---} \quad (i)$

At node 2

$i_2 = i_3 + i_4$

$V_1 - V_2 = 4mA + \frac{V_2 - 0}{12}$

$12(V_1 - V_2) = 48 + V_2$

$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V - 13V_2 \quad \text{---} \quad (ii)$

Solving V_1 and V_2 simultaneously, we have

$V_1 = 9.5V$ and $V_2 = 5.1V$

i.e Current through the 6Ω resistor

$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$; $i_2 = V_1 - V_2 = 9.5 - 5.1 = 4.4A$

Current through the 12Ω resistor

$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = 0.43A$

$V_1 = 9.5V$, $V_2 = 5.1V$

$i_1 = 1.58A$, $i_4 = 0.43A$