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**DEPARTMENT: COMPUTER SCIENCE 200level**

**Question 1**

A vector space over a real field F is a space that is closed under finite vector addition and scalar multiplication.

**Question 2**

A = (1, 1, 1)

B = (1, 2, 3)

C = (1, 5, 8)

αA + βB + γC = (a, b, c)

α 1 + β 1 + γ 1 = a

1 2 5 b

1 3 8 c

α + β + γ = a ……….. (1)

α + 2β + 5γ = b ………… (2)

α + 3β + 8γ = c …………. (3)

From equ (1)

α = a – β – γ ……….. (4)

Put equ (4) into equ (2) and equ (3)

**For equ (2)**

a – β – γ + 2β + 5γ = b

a + β + 4γ = b

β + 4γ = b – a ……… (5)

**For equ (3)**

a – β – γ + 3β + 8γ = c

a + 2β + 7γ = c

2β + 7γ = c – a ……. (6)

Combining equ (5) and equ (6)

β + 4γ = b – a ………. (multiply by 2)

2β + 7γ = c – a

2β + 8γ = 2b – 2a

- 2β + 7γ = c – a

γ = 2b – 2a –(c – a)

γ = 2b – 2a – c + a

γ = 2b – a – c

γ = -a +2b – c

From equ (5)

β = b – a - 4γ

β = b – a – 4(2b – a – c)

β = b – a – 8b + 4a + 4c

β = 3a – 7b + 4c

From equ (4)

α = a – β – γ

α = a – (3a – 7b + 4c) – (2b – a – c)

α = a – 3a + 7b – 4c – 2b + a + c

α = 5b – a – 3c

α = -a + 5b – 3c

αA + βB + γC

(–a + 5b – 3c) α + (3a – 7b + 4c) β + (-a +2b – c) γ

**Question 3**

P = (1, 2, 3)

Q = (3, 2, 1)

R = (0, 0, 1)

**Checking if the set is linearly independent**

Pα + Qβ +Rγ = 0

α 1 + β 3 + γ 0 = 0

2 2 0 0

3 1 1 0

α + 3β = 0 ……… (1)

2α + 2β = 0 ……… (2)

3α + β + γ = 0 …….. (3)

From equ (1)

α = -3β

Putting α = -3β into equ (2) and equ (3)

**For equ (2)**

2(-3β) + 2β = 0

-6β + 2β = 0

-4β = 0

β = 0

**For equ (3) also putting β = 0**

3(-3β) + β + γ = 0

-9β + γ = 0

-9(0) + γ = 0

γ = 0

Put β = 0 into equ (1)

α + 3(0) = 0

α = 0

Since α = 0, β = 0, γ = 0, then the vectors are linearly independent

**Checking if the set spans V**

Pα + Qβ +Rγ = (a, b, c)

α 1 + β 3 + γ 0 = a

2 2 0 b

3 1 0 c

α + 3β = a ……… (1)

2α + 2β = b ………. (2)

3α + β = a ………… (3)

From equ (1)

α = a - 3β …………. (4)

Putting equ (4) into equ (2) and equ (3)

**For equ (2)**

2(a - 3β) + 2β = b

2a - 6β + 2β = b

-4β = b – 2a

β = - b – 2a

4

β = - b + 2a

4

**For equ (3) also putting β**

3(a - 3β) + β + γ = c

3a - 9β + β + γ = c

-8β + γ = c – 3a

-8 - b + 2a + γ = c – 3a

4

-2(-b + 2a) + γ = c – 3a

2b – 4a + γ = c – 3a

γ = a – 2b + c

From equ (4)

α = a - 3β

α = a - 3 - b + 2a

4

α = a + 3b – 6a

4

α = 4a + 3b – 6a

4

α = -2a + 3b

4

-2a + 3b P + -b + 2a Q + (a – 2b + c )R

4 4

**Since the vectors spans R3 and are also linearly independent, then the vectors are basis R3.**