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COVID-19 HOLIDAY ASSIGNMENT

1. If A = $3i + 7j - 2k$, B = $i + 3j + 7k$, C = $9i - 4j + 6k$, find the angle between;
   (i) A and C;
   (ii) B and C;
   (iii) the unit vector in the direction of (A + B + C)

SOLUTION

(i) $\text{Cos}\,\Theta = \frac{\text{A.C}}{\|\text{A}\|\|\text{C}\|}$

$\text{Cos}\,\Theta = \frac{27 - 28 - 12}{\sqrt{3^2 + 7^2 + (-2)^2} \cdot \sqrt{9^2 + (-4)^2 + 6^2}}$

$= \frac{-13}{\sqrt{62} \cdot \sqrt{618}}$

$= \frac{-13}{7.87(24.86)}$

$= \frac{-13}{195.65}$

$\text{Cos}\,\Theta = -0.0664$

$\Theta = \text{Cos}^{-1}(-0.0664)$

$= 93.81°$

(ii) $\text{Cos}\,\Theta = \frac{\text{B.C}}{\|\text{B}\|\|\text{C}\|}$

$\text{Cos}\,\Theta = \frac{9 - 12 + 42}{\sqrt{(-1)^2 + 3^2 + 7^2} \cdot \sqrt{9^2 + (-4)^2 + 6^2}}$

$= \frac{39}{\sqrt{59} \cdot \sqrt{618}}$

$= \frac{39}{7.68(24.86)}$

$= \frac{39}{190.92}$

$\text{Cos}\,\Theta = 0.2043$

$\Theta = \text{Cos}^{-1}(0.2043)$

$= 78.21°$

(iii) $(\vec{A} + \vec{B} + \vec{C}) = (3 + 1 + 9)i + (7 + 3 - 4)j + (-2 + 7 + 6)k$

let $\vec{R} = (\vec{A} + \vec{B} + \vec{C})$

$\therefore \vec{R} = 13i + 6j + 11k$

Magnitude of $\vec{R} = \sqrt{13^2 + 6^2 + 11^2}$

$= \sqrt{326}$

$= 18.1$

Unit vector in direction of $\vec{R} = \frac{1}{|\vec{R}|}\vec{R}$

$\hat{R} = \frac{1}{18.1}(13i + 6j + 11k)$

$\hat{R} = \frac{13}{18.1}i + \frac{6}{18.1}j + \frac{11}{18.1}k$

Thus, required unit vector $= \frac{13}{18.1}i + \frac{6}{18.1}j + \frac{11}{18.1}k$

$= 0.72i + 0.33j + 0.61k$

2. A particle moves along a curve, $x = -8t^2$, $y = t^2 - 4t$, $z = t + 1$, where t is time. Find the modulus of acceleration at t = 1

SOLUTION

$\vec{r} = xi + yj + zk$

$\vec{r} = (8t^2)i + (t^2 - 4t)j + (t + 1)k$

$\frac{d\vec{r}}{dt}$ = velocity = v

$\frac{d\vec{r}}{dt} = (16t)i + (2t - 4)j + k$

$\frac{d^2\vec{r}}{dt} = (16)i + (2)j$

$\therefore \frac{d^2\vec{r}}{dt^2}$ = Acceleration = $16i + 2j$

Thus, modulus = $\sqrt{16^2 + 2^2}$

$= \sqrt{256 + 4}$

$= \sqrt{300}$

$= 17.32$

3. If $A = 4i + 2j - 4k$, $B = 8i - 2j + k$, $C = i + 4j - 3k$, find the vector triple product $(A \times B) \times C$

SOLUTION

$(A \times B) \times C = A \times (B \times C) - B \times (A \times C)$

$A \times (B \times C)$

$(B \times C) = \begin{vmatrix} i & j & k \\ 8 & -2 & 1 \\ 1 & 4 & -3 \end{vmatrix}$

$(B \times C) = [(-2 \times -3) - 4]i - [(8 \times -3) - 1]j + [(8 \times 4) - 2]k$

$= 2i + 25j + 30k$

$A \times (B \times C) = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 2 & 25 & 30 \end{vmatrix}$

$A \times (B \times C) = [60 - (-100)]i - [120 - (-8)]j + [100 - 4]k$

$\therefore A \times (B \times C) = 160i - 128j + 96k$

$B \times (A \times C)$

$(A \times C) = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 1 & 4 & -3 \end{vmatrix}$

$(A \times C) = [(2 \times -3) - (-12)]i - [(4 \times -3) - (-4)]j + [(4 \times 4) - 2]k$

$= 6i + 8j + 14k$

$$B \times (A \times C) = \begin{vmatrix} i & j & k \\ 8 & -2 & 1 \\ 6 & 8 & 14 \end{vmatrix}$$

$B \times (A \times C) = [(-28) - 8]i - [112 - (6)]j + [64 - (-12)]k$

$\therefore B \times (A \times C) = -36i - 106j + 76k$

Thus, $(A \times B) \times C = A \times (B \times C) - B \times (A \times C)$

$= 160i - 128j + 96k - (-36i - 106j + 76k)$

$= 196i - 22j + 20k$