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Find the ~~line~~ point of the intersection of the following lines on the circle

(i) $x - y - 14 = 0$ and $x^2 + y^2 - 6x + 8y = 0$

Solution

$x^2 + y^2 - 6x + 8y = 0$

Using completing the square method.

$x^2 - 6x + y^2 + 8y = 0$

add half the coefficient of x and y squared to both sides

$x^2 - 6x + 3^2 + y^2 + 8y + 4^2 = 0 + 4^2 + 3^2$

$(x-3)^2 (y+4)^2 = 16 + 9$

$(x-3)^2 (y+4)^2 = 25 \ldots (i)$

Centre $= (3, -4)$ and $r = \sqrt{25} = 5_u$

From the linear equation;

$x - y - 14 = 0$; make y the subject of the formula

$y = x - 14 \ldots$ (ii)

Sub equ (ii) into eqn (i)

$(x-3)^2 ((x-14) + 4)^2 = 25$

$(x-3)^2 (x-10)^2 = 25$

$(x-3)(x-3)(x-10)(x-10) = 25$

$x^2 - 3x - 3x + 9 + x^2 - 10x - 10x + 100 = 25$

$x^2 - 6x + 9 + x^2 - 20x + 100 = 25$

$2x^2 - 26x + 109 = 25$

$2x^2 - 26x + 109 - 25 = 0$

$\frac{2x^2}{2} - \frac{26x}{2} + \frac{84}{2} = 0$

$x^2 - 13x + 42 = 0$

$x^2 - 6x - 7x + 42 = 0$

$(x^2 - 6x) - (7x - 42) = 0$

$x(x-6) - 7(x-6) = 0$

$(x-7)(x-6) = 0$

$x = 7$ and $x = 6$

Substitute x into the linear equation.

$x = 7$; $y = 7 - 14 = -7 = A = (7, 7)_u$

$x = 6$; $y = 6 - 14 = -8 = A = (6, 6)_u$

~~$\therefore A = (-7, 7)$; $B(-8, 6)$~~

$A = (7, -7)$; $B(6, -8)$

(b) $2x+y-10=0$ and $x^2+y^2+4x-6y=0$

Solution

$x^2+y^2+4x-6y=0$

$x^2+4x+y^2-6y=0$

From completing the square method

$(x^2+4x+4)+y^2+6y+9=4+9$

$(x+2)^2+(y+3)^2=13$ ...(i)

Centre $=(2,3)$ radius $=\sqrt{13}$.

From linear equation

$y=2x-10$ ...(ii)

Sub into (i).

$(x+2)^2+(2x-10+3)^2=13$

$(x+2)^2+(2x-7)^2=13$

$(x+2)(x+2)+(2x-7)(2x-7)=13$

$x^2+2x+2x+4+4x^2-14x-14x+49-13=0$

36 $x^2+4x+4+4x^2-28x+49-13=0$

$5x^2-24x+40=0$

Its impossible.

(c) $x-5y-2=0$ and $x^2+25y^2-6xy-16=0$

Solution

Given $x^2+25y^2-6xy-16=0$

$x^2+y(25y-6x)-16=0$

Given $x-5y-2=0$

$x=5y-2$

$(5y-2)^2+y(25y-6(5y-2))-16=0$

$25y^2-20y+4+y(25y-30y+12)-16=0$

$25y^2-20y+25y^2-30y^2+12y-12=0$

$20y^2-8y-12=0$ solving quadratically.

$(20y^2-20y)+(12y-12)=0$

$20y(y-1)+12(y-1)=0$

$(20y+12)(y-1)=0$

$20y(y-1)+12(y-1)=0$

$\therefore 20y(y-1)=0$

$(20y-12)(y-1)$

$20y + 12 = 0$ or $y = 1$

$\frac{20y}{20} = \frac{-12}{20}$ or $y = 1$

$y = -3/5$ or $y = 1$

using the values find the points of interception

when $y = 3/5$ from (ii) $x = 5y - 2 = 5\left(\frac{-3}{5}\right) - 2 = -5$

Point $A = (-5, -3/5)$

when $y = 1$; from (ii) $x = 5y - 2 = 5(1) - 2 = 3$

Point $B = (3, 1)$

$\therefore$ The points will be $A(-5, -3/5)$ and $B(3, 1)$