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(1) If $\overline{M} = P\hat{i} - 6\hat{j} - 3\hat{k}$, $\overline{N} = 4\hat{i} + 3\hat{j} - \hat{k}$; $\overline{O} = \hat{i} - 3\hat{j} + 2\hat{k}$

find the value of P for which

(a) $\overline{M}$ and $\overline{N}$ are perpendicular to each other

Solution.

Given $M = P\hat{i} - 6\hat{j} - 3\hat{k}$ and $N = 4\hat{i} - 3\hat{j} - \hat{k}$

For perpendicular vectors, $\overline{M} \cdot \overline{N} = 0$

$(P\hat{i} - 6\hat{j} - 3\hat{k})(4\hat{i} + 3\hat{j} - \hat{k}) = 0$

$4P - 18 + 3 = 0$

$4P - 15 = 0$; $4P = 15$

$P = 15/4$ for perpendicular vectors

(b) $\overline{M}$, $\overline{N}$ and $\overline{O}$ are coplanar.

coplanar vectors or parallel vectors occurs when $\overline{M} \cdot (\overline{N} \times \overline{O}) = 0$

since $\overline{M} = P\hat{i} - 6\hat{j} - 3\hat{k}$; $\overline{N} = 4\hat{i} + 3\hat{j} - \hat{k}$;

$\overline{O} = \hat{i} - 3\hat{j} + 2\hat{k}$.

$$\overline{M} \cdot (\overline{N} \times \overline{O}) = \begin{vmatrix} P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$P \begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix} = 0$$

$P(6 - 3) + 6(8 + 1) - 3(-12 - 3) = 0$

$P(3) + 6(9) - 3(-15) = 0$

$3P + 54 + 45 = 0$

$\frac{3P}{3} = \frac{-\cancel{99}^{\,33}}{\cancel{3}_{\,1}}$

$\therefore P = -33$ when $\overline{M}$, $\overline{N}$ and $\overline{O}$ are parallel

(2.) Find the direction cosines and unit vector along the of $3\hat{i} + 2\hat{j} + 5\hat{k}$; $2\hat{i} - \hat{j} + 6\hat{k}$ and $5\hat{i} + 2\hat{j} - 3\hat{k}$

Solution.

Let $\overline{r}$ be the sum of $3\hat{i} + 2\hat{j} + 5\hat{k}$, $2\hat{i} - \hat{j} + 6\hat{k}$ and $5\hat{i} + 2\hat{j} - 3\hat{k}$.

$\overline{r} = (3\hat{i} + 2\hat{j} + 5\hat{k}) + (2\hat{i} - \hat{j} + 6\hat{k}) + (5\hat{i} + 2\hat{j} - 3\hat{k})$

$\bar{r} = 10\hat{i} + 3\hat{j} + 8\hat{k}$

(4) direction cosine is given as $\cos\alpha, \cos\beta, \cos\gamma$

$|r| = \sqrt{10^2 + 3^2 + 8^2}$

$|r| = \sqrt{100 + 9 + 64}$

$|r| = \sqrt{173} = 13.15$

$a_x = 10 \quad a_y = 3 \; ; \; a_z = 8$

$\cos\alpha = \frac{a_x}{|r|} = \frac{10}{13.15} = 0.7605$

$\cos\beta = \frac{a_y}{|r|} = \frac{3}{13.15} = 0.2281$

$\cos\gamma = \frac{a_z}{|r|} = \frac{8}{13.15} = 0.6084$

(5) The unit vector $= \frac{\bar{r}}{|r|} = \frac{10\hat{i} + 3\hat{j} + 8\hat{k}}{13.15}$

$\therefore$ The unit vector is $= \frac{10}{13.15}\hat{i} + \frac{3}{13.15}\hat{j} + \frac{8}{13.15}\hat{k}$

(8) If $f = 3u\hat{i} + u^2\hat{j} + (u+2)\hat{k}$ and $V = 2u\hat{i} - 3u\hat{j} + (u-2)\hat{k}$ evaluate the integral of $(f \times V)\,du$ from 0 to 1

given $f = 3u\hat{i} + u^2\hat{j} + (u+2)\hat{k}$ and

$V = 2u\hat{i} - 3u\hat{j} + (u-2)\hat{k}$

$$f \times V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix} \hat{i}\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - \hat{j}\begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + \hat{k}\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$\hat{i}\left(u^2(u-2) - (-3u(u+2))\right) - \hat{j}\left(3u(u-2) - 2u(u+2)\right)$
$+ \hat{k}\left(3u \cdot -3u - 2u \cdot u^2\right)$

$\hat{i}(u^3 - 2u^2 + 3u^2 + 6u) - \hat{j}(3u^2 - 6u - 2u^2 - 4)$
$+ \hat{k}(-9u^2 - 2u^3)$

$= (u^3 - u^2 + 6u)\hat{i} - (u^2 - 10u)\hat{j} + (-9u^2 - 2u^3)\hat{k} = f \times V$

Given $f \times V = (u^3 - u^2 - 6u)\hat{i} - (u^2 - 10u)\hat{j} + (-9u^2 - 2u^3)$

Therefore integrating

④ $\hat{i}\left[\frac{u^{3+1}}{4} + \frac{u^{2+1}}{3} + \frac{6u^2}{2}\right]_0^1 \Big| - \hat{j}\left[\frac{u^3}{3} - \frac{10u^3}{2}\right]_0^1 \Big| +$
$\hat{k}\left[-\frac{4}{3}u^3 - \frac{2u^4}{4}\right]_0^1 \Big|$

simplifying

$\hat{i}\left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right]_0^1 \Big| - \hat{j}\left[\frac{u^3}{3} - 5u^3\right]_0^1 \Big| +$
$\hat{k}\left[-\frac{4}{3}u^3 - \frac{u^4}{4}\right]_0^1 \Big|$

when $u = 1$

$\hat{i}\left[\frac{1^4}{4} + \frac{1^3}{3} + 3(1)^2\right]_0^1 \Big| - \hat{j}\left(\frac{1^3}{3} - 5(1)^3\right)_0^1 \Big| + \left[-\frac{4(1)}{3}\right.$
$\left.\frac{(1)^4}{4}\right]_0^1$

$\hat{i} = \frac{1}{4} + \frac{1}{3} + \frac{3}{1} = \frac{3+4+36}{12} = \frac{43}{12}\hat{i}$

$\hat{j} = \left[\frac{1}{3} - \frac{5}{1} = \frac{1-15}{3} = -\frac{14}{3}\hat{j}\right.$

$k = \left[-3(1)^2 - \frac{1^4}{2} = k . -3 - \frac{1}{2} = -\frac{7}{2}k\right.$

combining it

$\int \bar{F}x(u)du = \frac{43}{12}\hat{i} - \frac{14}{3}\hat{j} - \frac{7}{2}\hat{k}$