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Find the line point of the intersection of the following lines on the circle

① $x - y - 14 = 0$ and $x^2 + y^2 - 6x + 8y = 0$

Solution.

$$x^2 + y^2 - 6x + 8y = 0$$

Using completing the square method.

$$x^2 - 6x + y^2 + 8y = 0$$

add half the coefficient of x and y squared to both sides

$$x^2 - 6x + 3^2 + y^2 + 8y + 4^2 = 0 + 3^2 + 4^2$$

$$(x - 3)^2 + (y + 4)^2 = 16 + 9$$

$$(x - 3)^2 + (y + 4)^2 = 25 \dots (i)$$

Centre = $(3, -4)$ and $r = \sqrt{25} = 5$

From the linear equation:

$$x - y - 14 = 0; \text{ make } y \text{ the subject of the formula}$$

$$y = x - 14 \dots (ii)$$

Sub eqn (ii) into eqn (i)

$$(x - 3)^2 + (x - 14 + 4)^2 = 25$$

$$(x - 3)^2 + (x - 10)^2 = 25$$

$$(x - 3)(x - 3) + (x - 10)(x - 10) = 25$$

$$x^2 - 3x - 3x + 9 + x^2 - 10x - 10x + 100 = 25$$

$$x^2 - 6x + 9 + x^2 - 20x + 100 = 25$$

$$2x^2 - 26x + 109 = 25$$

$$2x^2 - 26x + 109 - 25 = 0$$

$$\frac{2x^2}{2} - \frac{26x}{2} + \frac{84}{2} = 0$$

$$x^2 - 13x + 42 = 0$$

$$x^2 - 6x - 7x + 42 = 0$$

$$(x^2 - 6x) - (7x - 42) = 0$$

$$x(x - 6) - 7(x - 6) = 0$$

$$(x - 7)(x - 6) = 0$$

$$x = 7 \text{ and } x = 6$$

Substitute x into the linear equation.

$$x = 7; y = 7 - 14 = -7 = A = (7, -7)$$

$$x = 6; y = 6 - 14 = -8 = A = (6, -8)$$

$$\therefore A = (7, -7); B = (6, -8)$$

$$A = (7, -7); B = (6, -8)$$

(b) $2x + y - 10 = 0$ and $x^2 + y^2 + cx + by = 0$

Solution

$$x^2 + y^2 + cx + by = 0$$

$$x^2 + cx + y^2 + by = 0$$

From completing the square method

$$(x^2 + cx + \frac{c^2}{4}) + y^2 + by + \frac{b^2}{4} = \frac{c^2}{4} + \frac{b^2}{4}$$

$$(x + \frac{c}{2})^2 + (y + \frac{b}{2})^2 = \frac{c^2 + b^2}{4}$$

Centre = $(-\frac{c}{2}, -\frac{b}{2})$ radius = $\frac{\sqrt{c^2 + b^2}}{2}$

From linear equation

$$y = 2x - 10 \dots (i)$$

sub into (ii).

$$(x+2)^2 + (2x-10+3)^2 = 13$$

$$(x+2)^2 + (2x-7)^2 = 13$$

$$(x+2)(x+2) + (2x-7)(2x-7) = 13$$

$$x^2 + 2x + 2x + 4 + 4x^2 - 14x - 14x + 49 - 13 = 0$$

$$x^2 + 4x + 4 + 4x^2 - 28x + 49 - 13 = 0$$

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$$5x^2 - 24x + 40 = 0$$

It's impossible.

(c) $x - 5y - 2 = 0$ and $x^2 + 25y^2 - 6xy - 16 = 0$

Solution

Given $x^2 + 25y^2 - 6xy - 16 = 0$

$$x^2 + y(25y - 6x) - 16 = 0$$

Given $x - 5y - 2 = 0$

$$x = 5y + 2$$

$$(5y+2)^2 + y(25y - 6(5y+2)) - 16 = 0$$

$$25y^2 - 20y + 4 + y(25y - 30y - 12) - 16 = 0$$

$$25y^2 - 20y + 25y^2 - 50y^2 + 12y - 12 = 0$$

$$20y^2 - 8y - 12 = 0 \text{ solving quadratically.}$$

$$(20y^2 - 20y)(y-1) = 0$$

$$20y(y-1) + 12(y-1) = 0$$

$$(20y + 12)(y-1) = 0$$

$$\therefore 20y(y-1) + 12(y-1) = 0$$

$$(20y - 12)(y-1) = 0$$

$$20y + 12 = 20 \quad \text{or } y = 1$$

$$\frac{20y}{20} = \frac{-12}{20} \quad \text{or } y = 1$$

$$y = -\frac{3}{5} \quad \text{or } y = 1$$

using the values find the points of intersection

when $y = \frac{3}{5}$ from (i) $x = 5y - 2 = 5\left(\frac{3}{5}\right) - 2 = 3 - 2 = 1$
Point A = $(1, \frac{3}{5})$

when $y = 1$; from (ii) $x = 5y - 2 = 5(1) - 2 = 3$
Point B = $(3, 1)$

\therefore the points will be A $(1, \frac{3}{5})$ and B $(3, 1)$