

Vector Space is a space consisting of vectors, together with the associative and commutative operation of addition of vectors, and the associative and distributive operation of multiplication of vectors by scalars.

②  $A = (1, 1, 1)$ ,  $B = (1, 2, 3)$ ,  $C = (1, 5, 8)$  spans  $\mathbb{R}^3$

$$\alpha A + \beta B + \gamma C = (a, b, c)$$

$$\begin{bmatrix} \alpha \\ \alpha \\ \alpha \end{bmatrix} + \begin{bmatrix} B \\ 2B \\ 3B \end{bmatrix} + \begin{bmatrix} \gamma \\ 5\gamma \\ 8\gamma \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\alpha + \beta + \gamma = a \quad \text{--- (1)}$$

$$\alpha + 2\beta + 5\gamma = b \quad \text{--- (2)}$$

$$\alpha + 3\beta + 8\gamma = c \quad \text{--- (3)}$$

From equation (1)

$$\alpha = a - \beta - \gamma \quad \text{--- (4)}$$

Put (4) in (2) and (3)

$$\text{In (2)} \rightarrow (a - \beta - \gamma) + 2\beta + 5\gamma = b$$

$$a - \beta - \gamma + 2\beta + 5\gamma = b$$

$$a + \beta + 4\gamma = b$$

$$\beta + 4\gamma = b - a \quad \text{--- (5)}$$

Put  $\alpha$  in equation (3)

$$(a - \beta - \gamma) + 3\beta + 8\gamma = c$$

$$a - \beta - \gamma + 3\beta + 8\gamma = c$$

$$a + 2\beta + 7\gamma = c$$

$$2\beta + 7\gamma = c - a \quad \text{--- (6)}$$

Combine (5) & (6)

$$\beta + 4\gamma = b - a \quad \text{--- (5)} \times 2$$

$$2\beta + 7\gamma = c - a \quad \text{--- (6)} \times 1$$

$$2\beta + 7\gamma = 2b - 2a$$

$$2\beta + 7\gamma = c - a$$

$$\gamma = (2b - 2a) - (c - a)$$

$$\gamma = 2b - 2a - c + a$$

$$\gamma = -a + 2b - c$$

$$\beta + 4\gamma = b - a$$

$$\beta + 4(-a + 2b - c) = b - a$$

$$\beta - 4a + 8b - 4c = b - a$$

$$\beta = b - a + 4a - 8b + 4c$$

$$\beta = 3a - 7b + 4c$$

From equation (4)

$$\alpha = a - \beta - \gamma$$

$$\alpha = (a - \beta - \gamma) - (\beta + 4\gamma)$$

$$\alpha = a - 3\beta - 5\gamma$$

$$\alpha = -a + 3b - 3c$$

$$(-a + 3b - 3c)A + (3a - 7b + 4c)B +$$

$$(-a + 2b - c)C$$

$$\text{③ } P = (1, 2, 3), Q = (3, 2, 1), R = (0, 0, 1)$$

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 3\alpha \\ 2\beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\alpha + 3\beta + 0 = 0 \quad \text{--- (1)}$$

$$2\alpha + 2\beta + 0 = 0 \quad \text{--- (2)}$$

$$3\alpha + \beta + \gamma = 0 \quad \text{--- (3)}$$

From equation (1) and (2) Spanning Set

$$\alpha + 3\beta = 0 \quad \text{--- (2)}$$

$$2\alpha + 2\beta = 0 \quad \text{--- (1)}$$

$$2\alpha + 6\beta = 0$$

$$2\alpha + 2\beta = 0$$

$$2\alpha + 6\beta = 2a$$

$$2\alpha + 2\beta = b$$

$$4\beta = 2a - b$$

$$\beta = \frac{2a - b}{4}$$

From equation 1

$$\alpha + 3\beta = a$$

$$\alpha + 3(2a - b/4) = a$$

$$\alpha + (6a - 3b/4) = a$$

$$\alpha = a - 6a + \frac{3b}{4} = a - \frac{5b}{4}$$

$$\alpha = \frac{a - 5b}{4}$$

$$\alpha = \frac{4a - 6a - 3b}{4}$$

$$\alpha = \frac{-2a + 3b}{4}$$

$$3\left(\frac{-2a + 3b}{4}\right) + \frac{2a - b}{4} + \frac{\gamma}{1} = c$$

$$\frac{-6a + 9b}{4} + \frac{2a - b}{4} + \frac{\gamma}{1} = c$$

$$\frac{-6a + 9b + 2a - b + 4\gamma}{4} = c$$

$$\gamma = a - 2b + c$$

$$\left(\frac{-2a + 3b}{4}\right)p + \left(\frac{2a - b}{4}\right)q + (a - 2b + c)r$$

Linearity Independence

$$\text{From eqn (1)} \quad \alpha = -3\beta \quad \text{--- (4)}$$

Put eqn (4) in (2)

$$2(-3\beta) + 2\beta = 0$$

$$-6\beta + 2\beta = 0$$

$$-4\beta = 0$$

$$\beta = 0$$

Put eqn (4) in (3)

$$3(-3\beta) + \beta + \gamma = 0$$

$$-9\beta + \beta + \gamma = 0$$

$$-8\beta + \gamma = 0$$

Put  $\beta = 0$  in eqn (5)

$$-6(0) + \gamma = 0$$

$$0 + \gamma = 0$$

$$\gamma = 0$$

$$\therefore \alpha = 0$$