Sub γ into β

$\beta = -b + c - 3(2b - a - c)$

$\beta = -b + c - 6b + 3a + 3c$

$\beta = -7b + 3a + 4c$

~~Sub β & γ into~~ (iv)

~~$\alpha = a -$~~

Sub β & v into (iv)

$\alpha = a - (-7b + 3a + 4c) - (2b - a - c)$

$= a + 7b - 3a - 4c - 2b + a + c$

$= -a + 5b - 3c$

3. $P = (1, 2, 3)$

$Q = (3, 2, 1)$

$R = (0, 0, 1)$

Sol.

First check whether its linear or not

$$\alpha\begin{bmatrix}1\\2\\3\end{bmatrix} + \beta\begin{bmatrix}3\\2\\1\end{bmatrix} + \gamma\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$\alpha + 3\beta = 0$ ---------- (i)

$2\alpha + 2\beta = 0$ ---------- (ii)

$3\alpha + \beta + \gamma = 0$ ---------- (iii)

From Eqn (i)

$\alpha = -3\beta$ ---- (iv)

Sub (iv) into (ii) ~~& iii~~

$2(-3\beta) + 2\beta = 0$

$-6\beta + 2\beta = 0$

$-4\beta = 0$

$\beta = 0$

Sub β into (iv)
$\alpha = -3(0)$
$\alpha = 0$
Sub α & β into (iii)
$3(0) + 0 + \gamma = 0$
$0 + 0 + \gamma = 0$
$\gamma = 0$
$\alpha = 0, \beta = 0$ & $\gamma = 0$
$\therefore$ The vectors are linearly independent.

* Spanning Sets

$$\alpha\begin{bmatrix}1\\2\\3\end{bmatrix} + \beta\begin{bmatrix}3\\2\\1\end{bmatrix} + \gamma\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$$

$\alpha + 3\beta = a$ ---- (i)
$2\alpha + 2\beta = b$ ---- (ii)
$3\alpha + \beta + \gamma = c$ ---- (iii)
From Eqn (i)
$\alpha = a - 3\beta$ ---- (iv)
Sub (iv) into (ii)
$2(a - 3\beta) + 2\beta = b$
$2a - 6\beta + 2\beta = b$
$2a - 4\beta = b$
$4\beta = 2a - b$
$\beta = \frac{2a-b}{4}$
Sub β into (iv)
$\alpha = a - 3\left(\frac{2a-b}{4}\right)$
$\alpha = a - \frac{6a+3b}{4}$
$\alpha = \frac{4a - 6a + 3b}{4}$
$\alpha = \frac{-2a+3b}{4}$
Sub α & β into (iii)
$3\left(\frac{2a+3b}{4}\right) + \left(\frac{2a-b}{4}\right) + \gamma = c$
Multiply through by 4
~~4a~~
$3(-2a+3b) + (2a-b) + 4\gamma = 4c$
$-6a + 9b + 2a - b + 4\gamma = 4c$
$-4a + 8b + 4\gamma = 4c$
Divide through by 4
$-a + 2b + \gamma = c$
$\gamma = c + a - 2b$

Since the vectors are linearly independent & spans of R^3 so the vectors are basis of R^3.

Assignment

1. A vector space over a field (F) is a set that is closed under finite vector addition and scalar multiplication for V to be a vector space.

2. $A = (1,1,1)$
$B = (1,2,3)$
$C = (1,5,8)$

Sol

$$\alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \gamma \begin{bmatrix} 1 \\ 5 \\ 8 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$= \alpha + \beta + \gamma = a$ — (i)
$\alpha + 2\beta + 5\gamma = b$ — (ii)
$\alpha + 3\beta + 8\gamma = c$ — (iii)

$\alpha = a - \beta - \gamma$ — (iv)

Sub (iv) into (ii) & (iii)

$(a-\beta-\gamma) + \beta + \gamma = b$
$a - \beta - \gamma + \beta + \gamma = b$
$a + \beta + 4\gamma = b$

$(a-\beta-\gamma) + 3\beta + 8\gamma = c$
$a - \beta - \gamma + 3\beta + 8\gamma = c$
$a + 2\beta + 7\gamma = c$

$a + \beta + 4\gamma = b$ — (v)
$a + 2\beta + 7\gamma = c$ — (vi)

$-\beta - 3\gamma = b - c$

$-\beta = b - c + 3\gamma$
$\beta = -b + c - 3\gamma$

Sub β into (v)

$a + (-b + c - 3\gamma) + 4\gamma = b$
$a - b + c - 3\gamma + 4\gamma = b$
$a - b + c + \gamma = b$
$\gamma = b - a + b - c$
$\gamma = 2b - a - c$

Sub γ into β