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ASSignment

1. A Vector Space over a field (F) is a set that is closed under finite vector addition and Scalar multiplication for V to be a vector Space.

2. $A = (1,1,1)$
$B = (1,2,3)$
$C = (1,5,8)$

Sol

$$\alpha\begin{pmatrix}1\\1\\1\end{pmatrix} + \beta\begin{pmatrix}1\\2\\3\end{pmatrix} + \gamma\begin{pmatrix}1\\5\\8\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix}$$

$= \alpha + \beta + \gamma = a$ — (i)
$\alpha + 2\beta + 5\gamma = b$ — (ii)
$\alpha + 3\beta + 8\gamma = c$ — (iii)

$\alpha = a - \beta - \gamma$ — (iv)

Sub (iv) Into (ii) & (iii)

$(a - \beta - \gamma) + \beta + \gamma = b$
$a - \beta - \gamma + \beta + \gamma = b$
$a + \beta + 4\gamma = b$

$(a - \beta - \gamma) + 3\beta + 8\gamma = c$
$a - \beta - \gamma + 3\beta + 8\gamma = c$
$a + 2\beta + 7\gamma = c$

$a + \beta + 4\gamma = b$ — (v)
$a + 2\beta + 7\gamma = c$ — (vi)

$-\beta - 3\gamma = b - c$

$-\beta = b - c + 3\gamma$
$\beta = -b + c - 3\gamma$

Sub β Into (v)
$a + (-b + c - 3\gamma) + 4\gamma = b$
$a - b + c - 3\gamma + 4\gamma = b$
$a - b + c + \gamma = b$
$\gamma = b - a + b - c$
$\gamma = 2b - a - c$

Sub γ Into β

Sub γ Into β
$\beta = -b + c - 3(2b - a - c)$
$\beta = -b + c - 6b + 3a + 3c$
$\beta = -7b + 3a + 4c$

Sub ~~β & γ Into~~ (iv)
~~$\alpha = a -$~~
Sub β & v Into (iv)
$\alpha = a - (-7b + 3a + 4c) - (2b - a - c)$
$= a + 7b - 3a - 4c - 2b + a + c$
$= -a + 5b - 3c$

3. $P = (1,2,3)$
$Q = (3,2,1)$
$R = (0,0,1)$

Sol.

First check whether its linear or not

$$\alpha\begin{pmatrix}1\\2\\3\end{pmatrix} + \beta\begin{pmatrix}3\\2\\1\end{pmatrix} + \gamma\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$\alpha + 3\beta = 0$ — (i)
$2\alpha + 2\beta = 0$ — (ii)
$3\alpha + \beta + \gamma = 0$ — (iii)

From Eqn (i)
$\alpha = -3\beta$ — (iv)

Sub (iv) Into (ii) ~~& (iii)~~
$2(-3\beta) + 2\beta = 0$
$-6\beta + 2\beta = 0$
$-4\beta = 0$
$\beta = 0$

Sub β Into (iv)
$\alpha = -3(0)$
$\alpha = 0$

Sub α & β Into (iii)
$3(0) + 0 + \gamma = 0$
$0 + 0 + \gamma = 0$
$\gamma = 0$

$\alpha = 0, \beta = 0 \;\&\; \gamma = 0$
$\therefore$ The vectors are linearly Independent.

* Spanning Sets

$$\alpha\begin{pmatrix}1\\2\\3\end{pmatrix} + \beta\begin{pmatrix}3\\2\\1\end{pmatrix} + \gamma\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix}$$

$\alpha + 3\beta = a$ — (i)
$2\alpha + 2\beta = b$ — (ii)
$3\alpha + \beta + \gamma = c$ — (iii)

From Eqn (i)
$\alpha = a - 3\beta$ — (iv)

Sub (iv) Into (ii)
$2(a - 3\beta) + 2\beta = b$
$2a - 6\beta + 2\beta = b$
$2a - 4\beta = b$
$4\beta = 2a - b$
$\beta = \frac{2a - b}{4}$

Sub β Into (iv)
$\alpha = a - 3\left(\frac{2a - b}{4}\right)$

$\alpha = a - \frac{6a + 3b}{4}$

$\alpha = \frac{4a - 6a + 3b}{4}$

$\alpha = \frac{-2a + 3b}{4}$

Sub α & β Into (iii)

$3\left(\frac{2a + 3b}{4}\right) + \left(\frac{2a - b}{4}\right) + \gamma = c$

Multiply through by 4

~~$4a +$~~
$3(-2a + 3b) + (2a - b) + 4\gamma = 4c$
$-6a + 9b + 2a - b + 4\gamma = 4c$
$-4a + 8b + 4\gamma = 4c$

Divide through by 4

$-a + 2b + \gamma = c$
$\gamma = c + a - 2b$

Since the vectors are linearly Independent & Spans of R^3 So the vectors are basis of R^3.