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18/SCI01/074

1. Define a vector space

A vector space over a real field F is a set that is closed under finite vector addition and scalar multiplication.

2. Show that the vectors A =(1, 1, 1) , B =(1, 2, 3), C =(1, 5, 8) spans R^3 .

Solution

$\alpha A + \beta B + \delta C$

$$\alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \delta \begin{pmatrix} 1 \\ 5 \\ 8 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$\alpha + \beta + \delta = a$ – (1)
$\alpha + 2\beta + 5\delta = b$ – (2)
$\alpha + 3\beta + 8\delta = c$ – (3)

from eqn (1)
$\alpha = a - \beta - \delta$ – (4)

put (4) in (2) and (3)
$\alpha + 2\beta + 5\delta = b$
$(a - \beta - \delta) + 2\beta + 5\delta = b$
$a - \beta - \delta + 2\beta + 5\delta = b$
$a + \beta + 4\delta = b - a$ –(5)

$\alpha + 3\beta + 8\delta = c$
$(a - \beta - \delta) + 3\beta + 8\delta = c$
$a - \beta - \delta + 3\beta + 8\delta = c$
$a + 2\beta + 7\delta = c - a$ –(6)

combine equation (5) and (6)

$\beta + 4\delta = b - a$ – (5)
-
$2\beta + 7\delta = c - a$ – (6)

Multiply equation (5) by 2 and equation (6) by 1

$$= \quad 2\beta + 8\delta = 2b - 2a$$
$$\underline{- 2\beta + 7\delta = c - a}$$
$$\delta = (2b - 2a) - (c - a)$$

$$\delta = 2b - 2a - c + a$$
$$\delta = 2b - a - c$$
$$\delta = -a + 2b - c$$

From equation (5)

$\beta + 4\delta = b - a$

Substitute the value of δ

$= \beta + 4(-a + 2b - c) = b - a$

$\beta - 4a + 8b - 4c = b - a$

$\beta = 4a - 8b + 4c = b - a$

$\beta = 3a - 7b + 4c$

From equation (4)

$\alpha = a - \beta - \delta$

$\alpha = a - (3a - 7b + 4c) - (-a + 2b - c)$

$\alpha = a - 3a + 7b - 4c + a - 2b + c$

$= -a + 5b - 3c$

3. Are the vectors A = (1, 2, 3), Q = (3, 2, 1), R = (0, 0, 1) a basis for R^3?

<u>Solution</u>

First check for linear dependency

$$\alpha P + \beta Q + \delta R$$

$$\alpha \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\alpha + 3\beta = 0 \quad - (1)$

$2\alpha + 2\beta = 0 \quad - (2)$

$3\alpha + \beta + \delta = 0 \quad - (3)$

From eqn (1)

$\alpha + \beta = 0$

Substitute eqn (1) in (2) and (3)

$2\alpha + 2\beta = 0$

$2(-3\beta) + 2\beta = 0$

$-6\beta + 2\beta = 0$

$-4\beta = 0$

$\beta = 0$

Substitute values of β into (2)

$\alpha + 3\beta = 0$

$\alpha + 0 = 0$

$\alpha = 0$

Substitute α and β into equation (3)

$3\alpha + \beta + \delta = 0$

$3(0) + 0 + \delta = 0$

$\delta = 0$

$\alpha + 3\beta = p \quad -(1)$

$2\alpha + 2\beta = q \quad -(2)$

$3\alpha + \beta + \delta = r \quad -(3)$

Combine equations (1) and (2)

$\alpha + 3\beta \quad = p \quad x1$

$\underline{2\alpha + 2\beta \quad = q \quad x2}$

Multiply equations (1) by 2 and (2) by 1

$2\alpha + 6\beta = 2p$

$\underline{- 2\alpha + 2\beta = q}$

$$\frac{4\beta}{4} = \frac{2p - q}{4}$$

$$\beta = \frac{2p - q}{4}$$

Substitute value of β into (1)

$\alpha + 3\beta = p$

$$\alpha + 3\left(\frac{2p - q}{4}\right) = p$$

$$\alpha + \left|\frac{6p - 3q}{4}\right| = p$$

$$\alpha = \frac{p}{1} - \left(\frac{6p - 3q}{4}\right)$$

$$\alpha = \frac{4p - (6p - 3q)}{4}$$

$$\alpha = \frac{4p - 6p + 3q}{4}$$

$$\alpha = \frac{-2p + 3q}{4}$$

In equation (2)

$3\alpha + \beta + \delta = r$

$\delta = r - 3\alpha - \beta$

$$\delta = \frac{r}{1} - 3\left(\frac{-2p + 3q}{4}\right) - \left(\frac{2p - q}{4}\right)$$

$$\delta = \frac{4r - 3(-2p + 3q) - (2p - q)}{4}$$

$$\delta = \frac{4r + 6p - 9q - 2p + q}{4}$$

$$\delta = \frac{4r + 4p - 8q}{4}$$

$$\delta = \frac{4p - 8q + 4r}{4}$$