* **NON LINEAR TRANSFORMATION**
* **A TRANSFORMATION DEFINED BY FORMULAR**
* **VERIFYING LINEARITY : ROTATION**
* **VERIFYING LINEARITY : DILATION**

**EXAMPLES**

**1 Non linear transformation**

Define T:R2→R2 by T(x)=1.5x. Verify that T is linear.

**Solution**

We have to check the defining properties for *all* vectors u,v and *all* scalars c. In other words, we have to treat u,v, and c as *unknowns*. The only thing we are allowed to use is the definition of T.

T(u+v)=1.5(u+v)=1.5u+1.5v=T(u)+T(v)T(cu)=1.5(cu)=c(1.5u)=cT(u).

Since T satisfies both defining properties, T is linear.

*Note:* we know from this example in Section 3.1 that T is a matrix transformation: in fact,

T(x)=A1.5001.5Bx.

Since a matrix transformation is a linear transformation, this is another proof that T is linear.

**2 transformation defined by formular**

Define T:R2→R3 by the formula

TExyF=C3x−yyxD.

Verify that T is linear.

**Solution**

EEx1y1F+Ex2y2FF=TEx1+x2y1+y2F=C3(x1+x2)−(y1+y2)y1+y2x1+x2D=C(3x1−y1)+(3x2−y2)y1+y2x1+x2D=C3x1−y1y1x1D+C3x2−y2y2x2D=TEx1y1F+TEx2y2F.

For the second property,

TEcEx1y1FF=TEcx1cy1F=C3(cx1)−(cy1)cy1cx1D=Cc(3x1−y1)cy1cx1D=cC3x1−y1y1x1D=cTEx1y1F.

Since T satisfies the defining properties, T is a linear transformation.

* **3 VERIFYING LINEARITY : ROTATION**
* Define T:R2→R2 by
* T(x)=thevectorxrotatedcounterclockwisebytheangleθ.
* Verify that T is linear.
* **Solution**
* Since T is defined geometrically, we give a geometric argument. For the first property, T(u)+T(v) is the sum of the vectors obtained by rotating u and v by θ. On the other side of the equation, T(u+v) is the vector obtained by rotating the sum of the vectors u and v. But it does not matter whether we sum or rotate first

**4 VERIFYING LINEARITY : DILATION**

Define T:R2→R2 by T(x)=1.5x. Verify that T is linear.

**Solution**

We have to check the defining properties for *all* vectors u,v and *all* scalars c. In other words, we have to treat u,v, and c as *unknowns*. The only thing we are allowed to use is the definition of T.

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