Name: Okafor Tochukwu Onyemilayo
Department: Mechanical Engineering
Matric No: 19/ENG06/045
Serial No: 108
Course: MAT104

Assignment

$y = \sin(6/x^2)$

$y + \Delta y = \sin\left(\frac{6}{(x+\Delta x)^2}\right)$

$\Delta y = \sin\left(\frac{6}{(x+\Delta x)^2}\right) - y$

$\Delta y = \sin\left(\frac{6}{(x+\Delta x)^2}\right) - \sin\left[\frac{6}{x^2}\right]$

$\Delta y = 2\cos\left[\frac{6x^2+6(x+\Delta x)^2}{2x^2(x+\Delta x)^2}\right]\sin\left[\frac{6x^2-6(x+\Delta x)^2}{2x^2(x+\Delta x)^2}\right]$

$\Delta y = 2\cos\left[\frac{3x^2+3(x+\Delta x)^2}{x^2(x+\Delta x)^2}\right]\sin\left[\frac{3x^2-3(x+\Delta x)^2}{x^2(x+\Delta x)^2}\right]$

$\Delta y = 2\cos\left[\frac{3x^2+3x^2+6x\Delta x+3(\Delta x)^2}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]\sin\left[\frac{3x^2-3x^2-6x\Delta x-3(\Delta x)^2}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]$

$\Delta y = 2\cos\left[\frac{6x^2+6x\Delta x+3(\Delta x)^2}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]\sin\left[\frac{-6x\Delta x-3(\Delta x)^2}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]$

$\frac{\Delta y}{\Delta x} = 2\cos\left[\frac{6x^2+6x\Delta x+3(\Delta x)^2}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]\sin\left[\frac{\Delta x(-6x-3\Delta x)}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]$

Δx

$\times$

$$\frac{\Delta y}{\Delta x} = 2\cos\left[\frac{6x^2+6x\Delta x+3(\Delta x)^2}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]\sin\left[\frac{\Delta x(-6x-3\Delta x)}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]\times\frac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2(\Delta x)^2}$$

$$\Delta x\times\left(\frac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right)$$

$$\frac{\Delta y}{\Delta x} = 2\left(\frac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right)\cos\left[\frac{6x^2+6x\Delta x+3(\Delta x)^2}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]\cdot\sin\left[\frac{\Delta x(-6x-3\Delta x)}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]$$

$$\Delta x\left(\frac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right)$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0} 2\left(\frac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right)\cos\left[\frac{6x^2+6x\Delta x+3(\Delta x)^2}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]\cdot\lim_{\Delta x\to 0}\sin\left[\frac{\Delta x(-6x-3\Delta x)}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right]$$

$$\Delta x\left(\frac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2(\Delta x)^2}\right)$$

$$\frac{\Delta y}{\Delta x} = 2\left(\frac{-6x-30}{x^4+2x^3(0)+x^2(0)^2}\right)\cos\left(\frac{6x^2+6x(0)+30^2}{x^4+2x^3(0)+x^2(0)^2}\right)\cdot 1$$

$$\frac{\Delta y}{\Delta x} = 2\left(\frac{-6x}{x^4}\right)\cos\left(\frac{6x^2}{x^4}\right)$$

$$\frac{\Delta y}{\Delta x} = 2\left(\frac{-6}{x^3}\right)\cos\left(\frac{6}{x^2}\right)$$

$$\frac{\Delta y}{\Delta x} = \frac{-12\cos(6/x^2)}{x^3}$$

2) $x = 4t^3 - t^2$

$y = t^4 + 2t^2$

$A = \int_a^b y\,dx$

$\frac{dx}{dt} = 12t^2 - 2t$

$dx = (12t^2 - 2t)\,dt$

$A = \int_1^3 (12t^2 - 2t)(t^4 + 2t^2)\,dt$

$= \int_1^3 (12t^6 + 24t^4 - 2t^5 - 4t^3)\,dt$

$= \left[\frac{12t^7}{7} + \frac{24t^5}{5} - \frac{2t^6}{6} - \frac{4t^4}{4}\right]_1^3$

$= \left[\frac{12t^7}{7} + \frac{24t^5}{5} - \frac{t^6}{3} - t^4\right]_1^3$

$= \left[\frac{12(3)^7}{7} + \frac{24(3)^5}{5} - \frac{(3)^6}{3} - 3^4\right] - \left[\frac{12(1)^7}{7} + \frac{24(1)^5}{5} - \frac{1^6}{3} - 1^4\right]$

$= \frac{26244}{7} - 12 + \frac{5832}{5} - 24 + \left(\frac{-729}{3} + 1\right) + (-81 + 1)$

$= \frac{26232}{7} + \frac{5808}{3} - \frac{728}{3} - 80$

$= 3747.43 + 1161.6 - 242.67 - 80$

$= 4586.36 \text{ unit}^2$

3) $x = 4t^3 - t^2$

$y = t^4 + 2t^2$

$\frac{dx}{dt} = 12t^2 - 2t$

$$\frac{dy}{dt} = 4t^3 + 4t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$= \frac{4t^3 + 4t}{12t^2 - 2t}$$

$$= \frac{2t(2t^2 + 2)}{2t(6t - 1)}$$

$$= \frac{2t^2 + 2}{6t - 1}$$