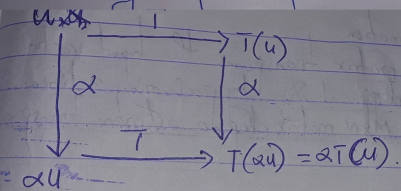
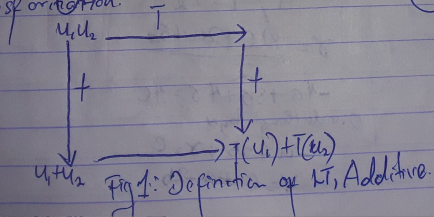


A linear transformation  $T: U \rightarrow V$  is a function that carries elements of vector space  $U$  (Domain) to vector space  $V$  (Co-domain) and which has two properties;

- (i)  $T(u_1 + u_2) = T(u_1) + T(u_2) \forall u_1, u_2 \in U$ .
- (ii)  $T(\alpha u) = \alpha T(u) \forall u \in U$  and  $\alpha \in F$  where  $F$  is a scalar field

NB: As every vector space properly derived from vector addition & scalar multiplication so to every property of linear transformation derives from any of these two properties. Below are two diagrams that convey the essence of the two defining properties of a linear transformation.



NB:  $T$  is the name of the linear transformation and should be used when we want to discuss the function as a whole.  $T(u)$  is how we talk about the output of the function of vector.

Example.

Given the matrix  $B = \begin{pmatrix} 4 & 5 & -3 \\ 2 & 5 & 1 \\ 3 & 2 & -1 \end{pmatrix}$  transform the vector  $\begin{pmatrix} 9 \\ 2 \\ 0 \end{pmatrix}$

Sol

$$T(x) = Bx$$

$$T(x) = \begin{pmatrix} 4 & 5 & -3 \\ 2 & 5 & 1 \\ 3 & 2 & -1 \end{pmatrix} \begin{pmatrix} 9 \\ 2 \\ 0 \end{pmatrix}$$

$$T(x) = 9 \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} - 2 \begin{pmatrix} 5 \\ 5 \\ 2 \end{pmatrix} + 0 \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

$$T(x) = \begin{pmatrix} 36 \\ 18 \\ 27 \end{pmatrix} + \begin{pmatrix} -10 \\ -10 \\ -4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$T(x) = \begin{pmatrix} 26 \\ 8 \\ 23 \end{pmatrix}. \text{ Hence, the transformation of } \begin{pmatrix} 9 \\ 2 \\ 0 \end{pmatrix} \text{ gives } \begin{pmatrix} 26 \\ 8 \\ 23 \end{pmatrix}.$$

Example  
Given that matrix  $A = \begin{pmatrix} 2 & -3 & 5 \\ -1 & 4 & 1 \\ 6 & 8 & 2 \end{pmatrix}$

transform the following vectors  
(i)  $\begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$  (ii)  $\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$  (iii)  $\begin{pmatrix} 8 \\ 3 \\ 1 \end{pmatrix}$  (iv)  $\begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$

Sol

$$(i) \quad TX = AX$$

$$TX = \begin{pmatrix} 2 & -3 & 5 \\ -1 & 4 & 1 \\ 6 & 8 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$$

$$TX = 2 \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix} + 5 \begin{pmatrix} -3 \\ 4 \\ 8 \end{pmatrix} - 1 \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -2 \\ 12 \end{pmatrix} + \begin{pmatrix} -15 \\ 20 \\ 40 \end{pmatrix} + \begin{pmatrix} -5 \\ -1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} -12 \\ 15 \\ 62 \end{pmatrix}. \text{ Hence, the transformation of } \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} \text{ gives } \begin{pmatrix} -12 \\ 15 \\ 62 \end{pmatrix}$$

$$(ii) \quad TX = AX$$

$$TX = \begin{pmatrix} 2 & -3 & 5 \\ -1 & 4 & 1 \\ 6 & 8 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

$$TX = 2 \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix} - 1 \begin{pmatrix} -3 \\ 4 \\ 8 \end{pmatrix} + 4 \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -2 \\ 12 \end{pmatrix} + \begin{pmatrix} 3 \\ -4 \\ -8 \end{pmatrix} + \begin{pmatrix} 20 \\ 4 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 27 \\ -2 \\ 12 \end{pmatrix}. \text{ Hence, the transformation of } \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \text{ gives } \begin{pmatrix} 27 \\ -2 \\ 12 \end{pmatrix}$$

$$(iii) \quad TX = AX$$

$$TX = \begin{pmatrix} 2 & -3 & 5 \\ -1 & 4 & 1 \\ 6 & 8 & 2 \end{pmatrix} \begin{pmatrix} 8 \\ 3 \\ 1 \end{pmatrix}$$

$$TX = 8 \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix} + 3 \begin{pmatrix} -3 \\ 4 \\ 8 \end{pmatrix} + 1 \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 16 \\ -8 \\ 48 \end{pmatrix} + \begin{pmatrix} -9 \\ 12 \\ 24 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 12 \\ 5 \\ 74 \end{pmatrix}. \text{ Hence, the transformation of } \begin{pmatrix} 8 \\ 3 \\ 1 \end{pmatrix} \text{ gives } \begin{pmatrix} 12 \\ 5 \\ 74 \end{pmatrix}$$

$$(iv) \quad TX = AX$$

$$TX = \begin{pmatrix} 2 & -3 & 5 \\ -1 & 4 & 1 \\ 6 & 8 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$$

$$TX = 6 \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix} - 3 \begin{pmatrix} -3 \\ 4 \\ 8 \end{pmatrix} + 2 \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 12 \\ -6 \\ 36 \end{pmatrix} + \begin{pmatrix} 9 \\ -12 \\ -24 \end{pmatrix} + \begin{pmatrix} 10 \\ 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 31 \\ -16 \\ 16 \end{pmatrix}. \text{ Hence, the transformation of } \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \text{ gives } \begin{pmatrix} 31 \\ -16 \\ 16 \end{pmatrix}$$