Vector space

(1) A space consisting of vectors together with the associative and commutative operation of addition of vectors, and the associative and distributive operation of multiplication of vectors by scalars

(2) Show that the vectors $A=(1,1,1)$ $B=(1,2,3)$ $C=(1,5,8)$ spans $\mathbb{R}^3$.

$$a\begin{pmatrix}1\\1\\1\end{pmatrix}+b\begin{pmatrix}1\\2\\3\end{pmatrix}+c\begin{pmatrix}1\\5\\8\end{pmatrix}=\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$a+b+c=x$$
$$a+2b+5c=y$$
$$a+3b+8c=z$$

Let A = coefficient matrix $\rightarrow$ $A=\begin{bmatrix}1&1&1\\1&2&5\\1&3&8\end{bmatrix}$

$$\det(A)=1\begin{bmatrix}2&5\\3&8\end{bmatrix}-1\begin{bmatrix}1&5\\1&8\end{bmatrix}+1\begin{bmatrix}1&2\\1&3\end{bmatrix}$$

$$1\left((8\times2)-(3\times5)\right)-1(8-5)+1[3-2]$$

$$1|16-15|$$

$$1 \quad -3 \quad + \quad 1$$

$$= -3 \neq 0 \rightarrow \det(A)\neq 0$$

the given vectors SPAN $\mathbb{R}^3$

(3) Are the vectors $P = (1,2,3)$, $Q = (3,2,1)$, $R = (0,0,1)$ a basis for $\mathbb{R}^3$?

$$P\begin{pmatrix}1\\2\\3\end{pmatrix} + Q\begin{pmatrix}3\\2\\1\end{pmatrix} + R\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$P + 3Q = x$$
$$2P + 2Q = y$$
$$3P + Q + R = z$$

let A = coefficient matrix $\rightarrow A = \begin{bmatrix}1 & 3 & 0\\ 2 & 2 & 0\\ 3 & 1 & 1\end{bmatrix}$

$$\det(A) = 1\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix} - 3\begin{bmatrix}2 & 0\\ 3 & 1\end{bmatrix} + 0\begin{bmatrix}2\\ 3\end{bmatrix}$$

$$1[2-0] - 3[2-0] + 0[1-2$$

$$0 - 0 - 0 = 0$$

$$= 0 \rightarrow \det(A) = 0$$

the given vector is not a bas
of $\mathbb{R}^3$