

# JOHN TISAN

18/SCI01/044

## MAT 204 ASSIGNMENT

1. Define a vector space

A vector space over a real field  $F$  is a set that is closed under finite vector addition and scalar multiplication.

2. Show that the vectors  $A=(1, 1, 1)$ ,  $B=(1, 2, 3)$ ,  $C=(1, 5, 8)$  spans  $\mathbb{R}^3$ .

Solution

$$\alpha A + \beta B + \delta C$$
$$\alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \delta \begin{pmatrix} 1 \\ 5 \\ 8 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\alpha + \beta + \delta = a \quad - (1)$$

$$\alpha + 2\beta + 5\delta = b \quad - (2)$$

$$\alpha + 3\beta + 8\delta = c \quad - (3)$$

from eqn (1)

$$\alpha = a - \beta - \delta \quad - (4)$$

put (4) in (2) and (3)

$$\alpha + 2\beta + 5\delta = b$$

$$(a - \beta - \delta) + 2\beta + 5\delta = b$$

$$a - \beta - \delta + 2\beta + 5\delta = b$$

$$a + \beta + 4\delta = b - a \quad - (5)$$

$$\alpha + 3\beta + 8\delta = c$$

$$(a - \beta - \delta) + 3\beta + 8\delta = c$$

$$a - \beta - \delta + 3\beta + 8\delta = c$$

$$a + 2\beta + 7\delta = c - a \quad - (6)$$

combine equation (5) and (6)

$$\beta + 4\delta = b - a \quad - (5)$$

-

$$2\beta + 7\delta = c - a \quad - (6)$$

---

Multiply equation (5) by 2 and equation (6) by 1

$$\begin{array}{r}
 = 2\beta + 8\delta = 2b - 2a \\
 - 2\beta + 7\delta = c - a \\
 \hline
 \delta = (2b - 2a) - (c - a)
 \end{array}$$

$$\delta = 2b - 2a - c + a$$

$$\delta = 2b - a - c$$

$$\delta = -a + 2b - c$$

From equation (5)

$$\beta + 4\delta = b - a$$

Substitute the value of  $\delta$

$$= \beta + 4(-a + 2b - c) = b - a$$

$$\beta - 4a + 8b - 4c = b - a$$

$$\beta = 4a - 8b + 4c = b - a$$

$$\beta = 3a - 7b + 4c$$

From equation (4)

$$\alpha = a - \beta - \delta$$

$$\alpha = a - (3a - 7b + 4c) - (-a + 2b - c)$$

$$\alpha = a - 3a + 7b - 4c + a - 2b + c$$

$$= -a + 5b - 3c$$

3. Are the vectors  $A = (1, 2, 3)$ ,  $Q = (3, 2, 1)$ ,  $R = (0, 0, 1)$  a basis for  $\mathbb{R}^3$ ?

Solution

First check for linear dependency

$$\alpha P + \beta Q + \delta R$$

$$\alpha \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\alpha + 3\beta = 0 \quad - (1)$$

$$2\alpha + 2\beta = 0 \quad - (2)$$

$$3\alpha + \beta + \delta = 0 \quad - (3)$$

From eqn (1)

$$\alpha + \beta = 0$$

Substitute eqn (1) in (2) and (3)

$$2\alpha + 2\beta = 0$$

$$2(-3\beta) + 2\beta = 0$$

$$-6\beta + 2\beta = 0$$

$$-4\beta = 0$$

$$\beta = 0$$

Substitute values of  $\beta$  into (2)

$$\alpha + 3\beta = 0$$

$$\alpha + 0 = 0$$

$$\alpha = 0$$

Substitute  $\alpha$  and  $\beta$  into equation (3)

$$3\alpha + \beta + \delta = 0$$

$$3(0) + 0 + \delta = 0$$

$$\delta = 0$$

$$\alpha + 3\beta = p \quad - (1)$$

$$2\alpha + 2\beta = q \quad - (2)$$

$$3\alpha + \beta + \delta = r \quad - (3)$$

Combine equations (1) and (2)

$$\alpha + 3\beta = p \quad \times 1$$

$$\underline{2\alpha + 2\beta = q \quad \times 2}$$

Multiply equations (1) by 2 and (2) by 1

$$\begin{array}{r} 2\alpha + 6\beta = 2p \\ - 2\alpha + 2\beta = q \\ \hline 4\beta = 2p - q \\ \frac{4\beta}{4} = \frac{2p - q}{4} \end{array}$$

$$\beta = \frac{2p - q}{4}$$

Substitute value of  $\beta$  into (1)

$$\alpha + 3\beta = p$$

$$\alpha + 3\left(\frac{2p - q}{4}\right) = p$$

$$\alpha + \left(\frac{6p - 3q}{4}\right) = p$$

$$\alpha = \frac{p}{1} - \left(\frac{6p - 3q}{4}\right)$$

$$\alpha = \frac{4p - (6p - 3q)}{4}$$

$$\alpha = \frac{4p - 6p + 3q}{4}$$

$$\alpha = \frac{-2p + 3q}{4}$$

In equation (2)

$$3\alpha + \beta + \delta = r$$

$$\delta = r - 3\alpha - \beta$$

$$\delta = \frac{r}{1} - 3 \left( \frac{-2p + 3q}{4} \right) - \left( \frac{2p - q}{4} \right)$$

$$\delta = \frac{4r - 3(-2p + 3q) - (2p - q)}{4}$$

$$\delta = \frac{4r + 6p - 9q - 2p + q}{4}$$

$$\delta = \frac{4r + 4p - 8q}{4}$$

$$\delta = \frac{4p - 8q + 4r}{4}$$

