

$$= \frac{2x^3 \ln x}{3} - \frac{2}{3} \cdot \frac{2x^2}{3} + C$$

$$= \frac{2x^3}{3} \left[\ln x - \frac{2}{3} \right] + C //$$

3) $x^2 \sin x dx$

soln

$$\text{let } u = x^2 \\ du = dx$$

$$\int dv = \int \sin x dx \\ v = -\cos x$$

using integration by parts.

$$\int u dv = uv - \int v du$$

$$= uv - \int v du$$

$$(x^2)(-\cos x) - \int (-\cos x)(dx)$$

$$= -x^2 \cos x + \int \cos x dx$$

$$= -x^2 \cos x + \sin x + C$$

4) $x \cos x dx$

soln

$$\text{let } u = x \\ du = dx$$

$$\int dv = \int \cos x dx \\ v = \sin x$$

using integration by parts

$$\int u dv = uv - \int v du$$

$$= uv - \int v du$$

$$(x)(\sin x) - \int (\sin x)(dx)$$

$$= x \sin x - (-\cos x) + C$$

$$= x \sin x + \cos x + C$$

Find the Integral of the Following

Soln

1) $\int e^x \sin x dx$

Using Integration by Parts.

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int e^x \sin x dx$$

$$\text{Let } u = e^x \quad \text{and} \quad \frac{dv}{dx} = \sin x$$

$$\frac{du}{dx} = e^x$$

$$v = -\cos x$$

$$\begin{aligned} \therefore \int e^x \sin x dx &= e^x(-\cos x) - \int (-\cos x)e^x dx \\ &= -e^x \cos x + \int e^x \cos x dx \end{aligned}$$

We have to find the integral of $\int e^x \cos x dx$.

$$\text{Let } u = e^x \quad \text{and} \quad \frac{dv}{dx} = \cos x$$

$$\frac{du}{dx} = e^x$$

$$v = +\sin x$$

$$\begin{aligned} \int e^x \cos x dx &= e^x \sin x - \int \sin x \cdot e^x dx \\ &= e^x \sin x - \int e^x \sin x dx \end{aligned}$$

$$1) \int e^x \sin x dx = -e^x \cos x + e^x \sin x - \int e^x \sin x dx$$

$$2) \int e^x \sin x dx = -e^x \cos x + e^x \sin x$$

$$2) \int e^x \sin x dx = e^x (\sin x - \cos x)$$

$$\therefore \int e^x \sin x dx = \frac{e^x}{2} (\sin x - \cos x) + C$$

2) $\int x^2 \ln x dx$

Soln

$$\int x^2 \ln x dx = \ln x \left[\frac{x^3}{3} \right] - \frac{2}{3} \int x^2 \cdot \frac{1}{x} dx$$

$$= \frac{x^3 \ln x}{3} - \frac{2}{3} \int x dx$$