A linear transformation is a function from one vector space to another that respects the underlying (linear) structure of each vector space

Examples

1 The linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ defined by $T(x, y, z) = (x - y, y - z)$ is given by the matrix

$$M = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

$$T(v) = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

(2) Is the linear transformation $T(x, y, z) = (x - y, y - z)$, from $\mathbb{R}^3$ to $\mathbb{R}^2$ injective?

For vector $v = (v_1, v_2, v_3)$ this can be written as

$$T(v) = Mv = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

iM is a 2x3 matrix so it is
Surjective because the minor $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$
has determinant $\underline{1}$