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COMPUTER ENGINEERING

19/ENGO2/066

1) $y = \sin\left(\frac{6}{x^2}\right)$

$y = \sin(6x^{-2})$

$y + \Delta y = \sin 6(x + \Delta x)^{-2}$

$y + \Delta y = \sin(6x^{-2} + \Delta x^{-2})$

$\Delta y = \sin(6x^{-2} + 6\Delta x^{-2}) - y$

$\Delta y = \sin(6x^{-2} + 6\Delta x^{-2}) - \sin 6x^{-2}$

$\dfrac{2\cos(A+B)}{2} \cdot \dfrac{\sin(A-B)}{2}$

$\Delta y = 2\cos\dfrac{(6x^{-2} + 6\Delta x^{-2} + 6x^{-2})}{2} \cdot \dfrac{\sin(6x^{-2} + 6\Delta x^{-2} - 6x^{-2})}{2}$

$\Delta y = 2\cos\dfrac{(12x^{-2} + 6\Delta x^{-2})}{2} \cdot \dfrac{\sin(6\Delta x^{-2})}{2}$

Divide through by Δx

$\dfrac{\Delta y}{\Delta x} = \dfrac{2\cos\dfrac{(12x^{-2} + 6\Delta x^{-2})}{2} \cdot \dfrac{\sin(6\Delta x^{-2})}{2}}{\Delta x}$

$= \dfrac{\cos(12x^{-2} + 6\Delta x^{-2})}{2} \cdot \dfrac{\sin(6\Delta x^{-2})}{2}$

$\lim_{\Delta x \to 0} \dfrac{\cos(12x^{-2} + 6\Delta x^{-2})}{2} \cdot \dfrac{\sin 6\Delta x^{-2}}{\frac{\Delta x}{2}}$

$\dfrac{dy}{dx} = \dfrac{\cos 12x^{-2}}{2} \cdot \dfrac{\Delta x}{2}$

$\dfrac{dy}{dx} = \cos 6x^{-2}$

2) $x = 4t^5 - t^2$ $y = t^4 + 2t^2$
   $t = 1$ and $i = 3$

$A = \int_a^b f(x)\,dx$ $y = f(x)$

$A = \int_a^b y\,dx$

Let

$x = f(t)$ and $y = g(t)$

$\dfrac{dx}{dt} = f'(t)$ and $dx = f'(t)\,dt$

$A = \int g(t)\, f'(t)\,dt$

$A = \int_1^3 (t^4 + 2t^2) \times (12t^2 - 2t)$

$A = \left[\dfrac{t^5}{5} + \dfrac{2t^2}{2}\right] \cdot \left[\dfrac{12t^3}{3} - \dfrac{2t^2}{2}\right]$

at $t = 1$

$A = \left[\dfrac{1^5}{5} + \dfrac{2}{2}\right] \times \left[\dfrac{12}{3} - \dfrac{2}{2}\right]$

$A = \left[\dfrac{1}{5} + 1\right] \times [4 - 1]$

$A = \dfrac{6}{5} \times 3$

$A = 18/5$

At $t = 3$

$A = \left[\dfrac{3^5}{5} + \dfrac{2(3)^2}{2}\right] \times \left[\dfrac{12(3)^3}{3} - \dfrac{2(3)}{2}\right]$

$A = \left[\dfrac{243}{5} + 9\right] \times [108 - 9]$

$A = (57.6) \times 99$

$A = 5702.4$

Upper − lower
$= 5702.4 - 18/5 = 5698.8 \text{ units}^2$

3) $x = 4t^3 - t^2$, $y = t^4 + 2t^2$

$$\frac{dx}{dt} = 12t^2 - 2t$$

$$\frac{dy}{dt} = 4t^3 + 4t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{4t^3 + 4t}{12t^2 - 2t}$$

$$\frac{dy}{dx} = \frac{4t^3}{3\sqrt{2t^4 - 2t}} + \frac{4t}{3\sqrt{2t^2 - 2t}}$$

$$\frac{dy}{dx} = \frac{1}{3 - 2} + \frac{1}{3t^2 - 2}$$

$$\frac{dy}{dx} = 1 + \frac{1}{3t^2 - 2} \; //$$