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MATRIC NO; 19/ENG02/053
COURSE; MAT 104

(1) Differentiate $y = \sin(6/x^2)$ from first principle.

Solution

$y = \sin(6x^{-2})$

$y + \Delta y = \sin 6(x+\Delta x)^{-2}$

$y + \Delta y = \sin(6x^{-2} + 6\Delta x^{-2})$

$\Delta y = \sin(6x^{-2} + 6\Delta x^{-2}) - \sin 6x^{-2}$

$2\cos\frac{(A+B)}{2} \cdot \sin\frac{(A-B)}{2}$

$2\cos\frac{(6x^{-2} + 6\Delta x^{-2} + 6x^{-2})}{2} \sin\frac{(6x^{-2} + 6\Delta x^{-2} - 6x^{-2})}{2}$

$= 2\cos\frac{(12x^{-2} + 6\Delta x^{-2})}{2} \sin\frac{(6\Delta x^{-2})}{2}$

divide both sides by $\Delta x$

$\frac{\Delta y}{\Delta x} = 2\cos\frac{(12x^{-2} + 6\Delta x^{-2})}{2} \cdot \sin\frac{(6\Delta x^{-2})}{2}$

$\lim_{\Delta x \to 0} = \cos\frac{(12x^{-2} + 6\Delta x^{-2})}{2} \cdot \frac{\sin\frac{(6\Delta x^{-2})}{2}}{\Delta x/2}$

$\lim_{\Delta x \to 0} = \frac{\cos(12x^{-2} + 6(0)^{-2})}{2} \quad \frac{\sin\frac{6\Delta x^{-2}}{2}}{\frac{\Delta x}{2}} \longrightarrow 1$

$\frac{\Delta y}{\Delta x} = \frac{\cos 12x^{-2}}{2}$

$\therefore \frac{\Delta y}{\Delta x} = \cos 6x^{-2}$

(2.) Find the area under the curve given parametric equations $x=4t^3-t^2$ $y=t^4+2t^2$ at $t=1$ and $t=3$

Soln.

Let A represent the area, then

$$A=\int_a^b y\,dx$$

$$y=t^4+2t^2$$

$$A=\int_a^b t^4+2t^2\,dx$$

Given that $x=4t^3-t^2$

$$\frac{dx}{dt}=12t^2-2t$$

$$\therefore dx=12t^2-2t\,dt$$

$$A=\int_1^3 t^4+2t^2(12t^2-2t\,dt)$$

$$A=\int_1^3 12t^6-2t^5+24t^4-4t^3\,dt$$

$$=\left[\frac{12t^7}{7}-\frac{2t^6}{6}+\frac{24t^5}{5}-\frac{4t^4}{4}\right]_1^3$$

$$=\left[\frac{12}{7}t^7-\frac{t^6}{3}+\frac{24t^5}{5}-t^4\right]_1^3$$

$$=\left[\frac{12(3)^7}{7}-\frac{(3)^6}{3}+\frac{24(3)^5}{5}-(3)^4\right]-\left[\frac{12}{7}-\frac{1}{3}+\frac{24}{5}-1\right]$$

$$=\frac{160704}{35}-\frac{544}{105}$$

$=$ 4586.36 sq units

1) $x = 4t^3 - t^2$ $y = t^4 + 2t^2$ find $\frac{dy}{dx}$

soln

$x = 4t^2 - t^2$

$\frac{dx}{dt} = 12t^2 - 2t$

$y = t^4 + 2t^2$

$\frac{dy}{dt} = 4t^3 + 4t$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= 4t^3 + 4t \times \frac{1}{12t^2 - 2t}$$

$$= \frac{4t^3 + 4t}{12t^2 - 2t}$$

$$= \frac{\not{4}^{2}(t^3 + t)}{\not{2}(6t^2 - t)}$$

$$= \frac{2(t^3 + t)}{6t^2 - t}$$

$$\therefore \frac{dy}{dx} = \frac{2t^3 + 2t}{6t^2 - t}$$