1) Differentiat $y = \sin\left(\frac{6}{x^2}\right)$ from first principal

Solution

$y = \sin(6x^{-2})$

$y + \Delta y = \sin 6(x + \Delta x)^{-2}$

$y + \Delta y = \sin(6x^{-2} + 6\Delta x^{-2})$

$\Delta y = \sin(6x^{-2} + 6\Delta x^{-2}) - \sin 6x^{-2}$

$2\cos\frac{(A+B)}{2} \cdot \sin\frac{(A-B)}{2}$

$$2\cos\left(\frac{6x^{-2} + 6\Delta x^{-2} + 6x^{-2}}{2}\right) \sin\left(\frac{6x^{-2} + 6\Delta x^{-2} - 6x^{-2}}{2}\right)$$

$$\cancel{\frac{2\cos(6x^{-2} + 6\Delta x^{-2} + 6x^{-2})}{\Delta x}}$$

$$= 2\cos\left(\frac{12x^{-2} + 6\Delta x^{-2}}{2}\right) \sin\left(\frac{6\Delta x^{-2}}{2}\right)$$

divide both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\left(\frac{12x^{-2} + 6\Delta x^{-2}}{2}\right) \sin\left(\frac{6\Delta x^{-2}}{2}\right)}{\Delta x}$$

$$\lim_{\Delta \to 0} = \cos\left(\frac{12x^{-2} + 6\Delta x^{-2}}{2}\right) \frac{\sin\left(\frac{6\Delta x^{-2}}{2}\right)}{\frac{\Delta x}{2}}$$

$$\lim_{\Delta \to 0} = \cos\left(\frac{12x^{-2} + 6(0)^{-2}}{2}\right) \quad \frac{\sin\frac{6\Delta x^{-2}}{2}}{\frac{\Delta x}{2}} \longrightarrow 1$$

$$\frac{\Delta y}{\Delta x} = \cos\left(\frac{\overset{6}{\cancel{12}} x^{-2}}{\cancel{2}}\right)$$

$$\frac{\Delta y}{\Delta x} = \cos 6x^{-2} \;//$$

2) find the area under the curve, given parametric equations $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$, at $t = 1$ and $t = 3$

Solution

given $y = t^4 + 2t^2$

$$A = \int_a^b y\,dx$$

$$A = \int_1^3 t^4 + 2t^2\,dx$$

Given $x = 4t^3 - t^2$

$$\frac{dx}{dt} = 12t^2 - 2t$$

$$dx = (12t^2 - 2t)\,dt$$

$$A = \int_1^3 t^4 + 2t^2(12t^2 - 2t)\,dt$$

$$A = \int_1^3 12t^6 - 2t^5 + 24t^4 - 4t^3\,dt$$

$$\left[\frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4}\right]$$

$$= \left[\frac{12(3)^7}{7} - \frac{2(3)^6}{6} + \frac{24(3)^5}{5} - \frac{4(3)^4}{4}\right] - \left[\frac{12(1)^7}{7} - \frac{2(1)^6}{6} + \frac{24(1)^5}{5} - \frac{4(1)^4}{4}\right]$$

$$\left[\frac{26244}{7} - 1053\right] - \cancel{\left[\frac{544}{495}\right]}\left[\frac{12}{7} - \frac{2}{6} + \frac{24}{5} - 1\right]$$

$$\left[\frac{18873}{7}\right] - \left[\frac{544}{105}\right]$$

$$= 2690.96$$

3) If $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$, find dy/dx

Solution

$$\frac{dx}{dt} = 12t^2 - 2t \qquad \frac{dy}{dt} = 4t^3 + 4t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{4t^3 + 4t}{12t^2 - 2t}$$