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COMPUTER ENGINEERING (19/ENG02/054)
MAT 104 Serial No : 130
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1) Differentiate $y = \sin(6/x^2)$ using first principle.

Soln

$$\frac{d}{dx}(f(g)) = \frac{d}{dg}(f(g)) \times \frac{d}{dx}(g)$$

where $g = 6/x^2$

$$\therefore \frac{d}{dx}\sin\left[\frac{6}{x^2}\right] = \frac{d}{dg}(\sin(g)) \times \frac{d}{dx}(6/x^2)$$

Using quotient rule, $\frac{d}{dx}\left(\frac{6}{x^2}\right) = -12/x^3$

and $\frac{d}{dg}(\sin(g)) = \cos(g)$

$$\cos\left[6/x^2\right] \times \left[-12/x^3\right]$$

$$\frac{d}{dx}\sin\left[\frac{6}{x^2}\right] = \frac{-12\cos(6/x^2)}{x^3}$$

$\cancel{= -12\cos(6/x^2)}$

(2) $x = 4t^3 - t^2$, $y = t^4 + 2t^2$ where $t = 1 \,\&\, 3$

$A = \int_a^b y\,dx$ $\quad y = t^4 + 2t^2$

$$A = \int_1^3 t^4 + 2t^2$$

Given that $x = 4t^3 - t^2$; $\dfrac{dx}{dt} = 12t^2 - 2t$

$$dx = (12t^2 - 2t) \cdot dt$$

$$A = \int_1^3 (t^4 + 2t^2) \times (12t^2 - 2t)$$

$$A = \int_1^3 t^4(12t^2 - 2t) + 2t^2(12t^2 - 2t)$$

$$A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3)\,dt$$

$$A = \int_1^3 \frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4}$$

$$A = \left[\frac{12(3)^7}{7} - \frac{2(3)^6}{6} + \frac{24(3)^5}{5} - \frac{4(3)^4}{4}\right] - \left[\frac{1^2}{7} - \frac{2}{6} + \frac{2^4}{5} - \frac{4}{4}\right]$$

$$A = \left[\frac{26244}{7} - 243 + 1166.4 - 81\right] - \left[\frac{544}{105}\right]$$

$$A = 2420.74 - 5.1809$$

$$A = 2,815.559$$

$A \simeq 2,815.6$ square units

(3) $x = 4t^3 - t^2$; $dx/dt = 12t^2 - 2t$

$y = t^4 + 2t^2$; $dy/dt = 4t^3 + 4t$

$dy/dx = dy/dt \div dx/dt = dy/dx \times dt/dx$

$$\frac{dy}{dx} = \frac{4t^3 + 4t}{12t^2 - 2t}$$