

① Question
Given the following

$$X = \begin{pmatrix} 1 & 2 & 8 \\ 4 & 7 & 6 \\ 9 & 5 & 3 \end{pmatrix} \quad \text{and} \quad Y = \begin{pmatrix} 0 & 5 & 0 \\ -3 & -7 & -1 \\ 2 & 1 & 9 \end{pmatrix}$$

Determine

- (i) Whether X is singular or non-singular
- (ii) Whether Y is singular or non-singular
- (iii) Rank of X
- (iv) Whether $(X+Y)$ is singular or non-singular
- (v) Whether XY is singular or non-singular.

Solution
i) Whether X is singular or non-singular

$$* = \begin{bmatrix} 1 & 2 & 8 \\ 4 & 7 & 6 \\ 9 & 5 & 3 \end{bmatrix}$$

$$*/ = \begin{vmatrix} 1 & 2 & 8 \\ 4 & 7 & 6 \\ 9 & 5 & 3 \end{vmatrix}$$

$$= 1 \times \begin{vmatrix} 7 & 6 \\ 5 & 3 \end{vmatrix} - 2 \times \begin{vmatrix} 4 & 6 \\ 9 & 3 \end{vmatrix} + 8 \times \begin{vmatrix} 4 & 7 \\ 9 & 5 \end{vmatrix}$$

$$= 1 \times (21 - 30) - 2 \times (12 - 54) + 8 \times (20 - 63)$$

$$= 1 \times (-9) - 2 \times (-42) + 8 \times (-43)$$

$$= -9 + 84 - 344$$

$$= -269$$

hence, $|X| \neq 0$ is non-singular matrix

(ii) Y

$$y = \begin{bmatrix} 0 & 5 & 0 \\ -3 & & \\ & & \end{bmatrix} \quad y = \begin{bmatrix} 0 & 5 & 0 \\ -3 & -7 & -1 \\ 2 & 1 & 9 \end{bmatrix}$$

$$|Y| = \begin{vmatrix} 0 & 5 & 0 \\ -3 & -7 & -1 \\ 2 & 1 & 9 \end{vmatrix}$$

$$0 \times \begin{vmatrix} -7 & -1 \\ 1 & 9 \end{vmatrix} - 5 \times \begin{vmatrix} -3 & -1 \\ 2 & 9 \end{vmatrix} + 0 \times \begin{vmatrix} -3 & -7 \\ 2 & 1 \end{vmatrix}$$

$$0 \times (-63 + 1) - 5 \times (-27 + 2) + 0 \times (-3 + 14)$$

$$0 \times (-62) - 5 \times (-25) + 0 \times 11$$

$$0 + 125 + 0$$

$$= 125 \quad \text{non singular}$$

(ii) Rank of Y

$$Y = \begin{bmatrix} 0 & 5 & 0 \\ -3 & -7 & -1 \\ 2 & 1 & 9 \end{bmatrix}$$

$$\text{Rank} \begin{bmatrix} 0 & 5 & 0 \\ -3 & -7 & -1 \\ 2 & 1 & 9 \end{bmatrix}$$

$$R_1 \leftarrow R_1 \div 3$$

$$= \begin{bmatrix} 1 & 2.33 & 0.33 \\ 0 & 5 & 0 \\ 2 & 1 & 9 \end{bmatrix}$$

$$R_3 \leftarrow R_3 - 2R_1$$

$$= \begin{bmatrix} 1 & 2.33 & 0.33 \\ 0 & 5 & 0 \\ 0 & -3.67 & 8.33 \end{bmatrix}$$

$$R_2 \leftarrow R_2 \div 5$$

$$\begin{bmatrix} 1 & 2.33 & 0.33 \\ 0 & 1 & 0 \\ 0 & -3.67 & 8.33 \end{bmatrix}$$

$$R_3 \leftarrow R_3 + 3.67R_2$$

$$\begin{bmatrix} 1 & 2.33 & 0.33 \\ 0 & 1 & 0 \\ 0 & 0 & 8.33 \end{bmatrix}$$

$$R_3 \leftarrow R_3 \times 0.12$$

$$\begin{bmatrix} 1 & 2.33 & 0.33 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The rank of the matrix
is the number of non-
all zeros rows

$$\text{Rank} = 3 =$$

Rank of $(X+Y)$

$$X+Y = \begin{bmatrix} 1 & 7 & 8 \\ 1 & 0 & 5 \\ 7 & 4 & -6 \end{bmatrix}$$

$$1 \times \begin{vmatrix} 0 & 5 \\ 4 & -6 \end{vmatrix} - 7 \begin{vmatrix} 1 & 5 \\ 7 & -6 \end{vmatrix} + 8 \times \begin{vmatrix} 1 & 0 \\ 7 & 4 \end{vmatrix}$$

$$1 \times (0 - 20) - 7 \times (-6 - 35) + 8 \times (4 + 0)$$

$$1 \times (-20) - 7 \times (-41) + 8 \times (4)$$

$$= -20 + 287 + 32$$

$$= 299$$

It is ~~is~~ non-singular matrix

(iv) $5Y$

$$Y = \begin{bmatrix} 0 & 5 & 0 \\ -3 & -7 & -1 \\ 2 & 1 & 9 \end{bmatrix}$$

Determine if $5Y$ is singular or non-singular matrix

$$5Y = \begin{bmatrix} 0 & 25 & 0 \\ -15 & -35 & -5 \\ 10 & 5 & 45 \end{bmatrix}$$

$$5Y = 0x \begin{vmatrix} -35 & -5 \\ 5 & 45 \end{vmatrix} - 25x \begin{vmatrix} -15 & -5 \\ 10 & 45 \end{vmatrix} + 0x \begin{vmatrix} -15 & -35 \\ 10 & 5 \end{vmatrix}$$

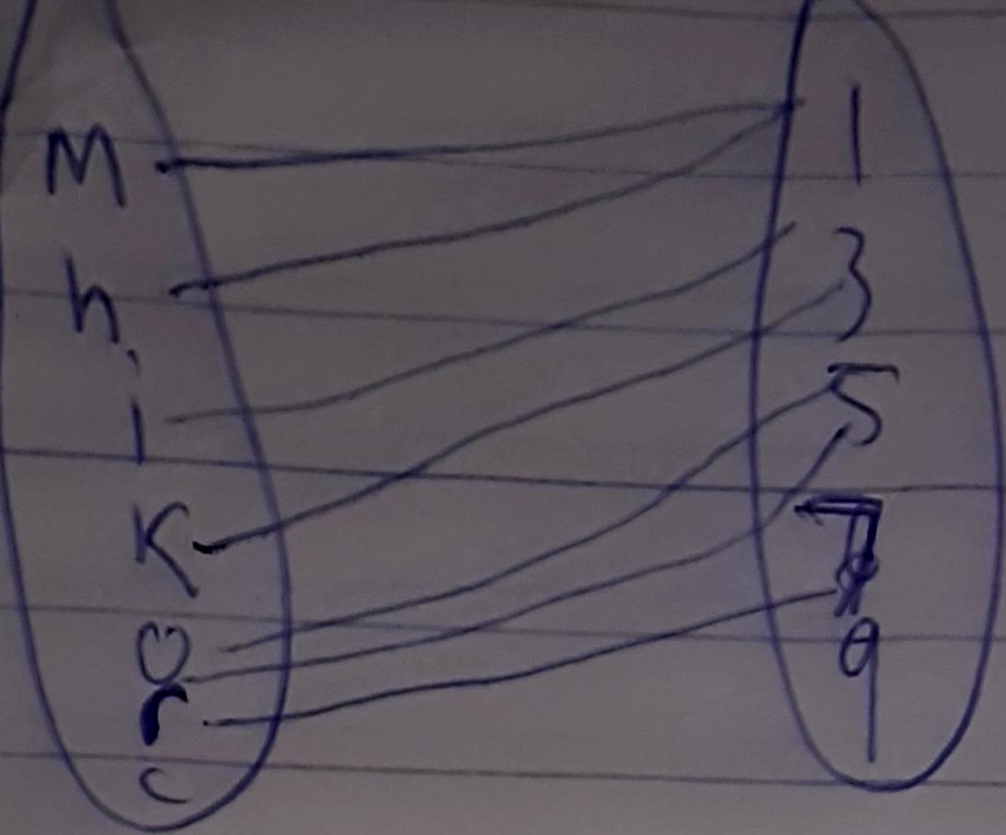
$$= 0x (-1575 + 25) - 25x (-675 + 50) + 0x (-75 + 350)$$

$$= 0x (-1550) - 25x (-625) + 0x (275)$$

$$= 0 + 15625 + 0$$

$$= 15625$$

Non singular



~~$T(A)$~~

$$T(M) = T(h) = 1$$

$$T(i) = T(k) = 3$$

$$T(0) = \cancel{T(*)} = 5$$

$$T(R) = 7$$