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17/Eng04/014

SYSTEM RESPONSE 1

① From the diagram

$\text{Spring} = K(x-0)$

$F\,\text{damper} = Kd \dfrac{d(x-0)}{dt}$

$f(t) \Rightarrow F(t)$

$F(t) - K(x-0) - Kd \dfrac{d(x-0)}{dt}$

Taking the Laplace Transform

$\therefore F(s) = Kx(s) - Kds\, x(s) = 0$

$F(s) = [K + Kds]\, x(s)$

$Gs = \dfrac{x(s)}{F(s)} = \dfrac{1}{K + Kds}$

$\Rightarrow \dfrac{1/K}{1 + (Kd/K)s}$ -

$\dfrac{x}{F(s)} = \dfrac{1/K}{(Kd/K)s + 1} = \dfrac{1/K}{Ts + 1}$

$T = \dfrac{Kd}{K}$

$T = \dfrac{0.03}{4 \times 10^{3}}$

$T = 7.5 \times 10^{-6}\, s$

X_0 after Tseconds

$X_0 = F/K\,[1 - e^{-1}]$

$X_0 = 100/4 \times 10^{-3}\,[1 - e^{-1}]$

$X_0 = 100/4 \times 10^{-3} \times 0.632$

$X_0 = 0.0158\, m$, $16 \times 10^{-3} = 16\, mm$ //

(2) $E_2 = M_c \Delta\theta \Rightarrow m_c [\theta_2 - \theta_1]$

$E_1 = m_c [\theta_2 - \theta_1]$

θ = Now Temp

$$G(s) = \frac{E_2}{E_1} = \frac{mc[\theta_2 - \theta_1]}{mc[\theta_2 - \theta_1]} = 1/Ts+1$$

$$\frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{1}{Ts+1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

let $\theta_2 - \theta_1 (s) = K(t)$

$$[\theta - \theta_1](s) = \frac{K(t)}{Ts+1}$$

Then

$$\theta - \theta_1 (s) = \frac{K(t)[1/T]}{s + 1/T}$$

Laplace transform of kd t

$= K/s$

$$(\theta - \theta_1](s) = \frac{K[1/T]}{s[s+1/T]}$$

$LT^{-1} [\theta - \theta_1] = K[1 - e^{-t/T}]$

$\theta(t) = \theta_1 + [\theta_2 - \theta_1][1 - e^{-t/T}$

$119 = 20 + [120 - 20][1 - e^{-t/T}]$.

$99 = 100[1 - e^{-t/T}]$

$0.99 = [1 - e^{-t/T}]$

$0.99 - 1 = -e^{-t/T}$

$\ln 0.01 = -t/T$

$T = 6/4.6045 = 1.302 \text{mins}$

$T = 78.17 \text{Sec}$.

Thermal capacitance $c = mc = 0.5 \times 34.6 = 173 \ J/K$.

$T = RC$, $R = T/C$

$R = \frac{78.17}{173} = 0.451 K/W$.

(3) $\frac{W}{Km\,x} = \frac{1}{Ts+1}$

$T = 1/K_3 \qquad Km = \frac{W K_2}{K_3}$

$W = \frac{Km\,x}{Ts+1}$

Laplace Transform of the step input.

$W = \frac{Km\,x}{s}\left[\frac{1}{Ts+1}\right].$

$\frac{Km\,x}{s}\left[\frac{1/T}{s+1/T}\right]$

$W(s) = Km\,x\left[1-e^{-t/T}\right]$

at $t=0$ $\quad Km\,x\left[1-e^{0}\right] = 1$

at $t=T$ $\quad Km\,x\left[1-e^{-1(t/T)}\right] = 0.63\,Km\,x$

at $t = 4T$ $= Km\,x\left[1-e^{-4T/t}\right]$

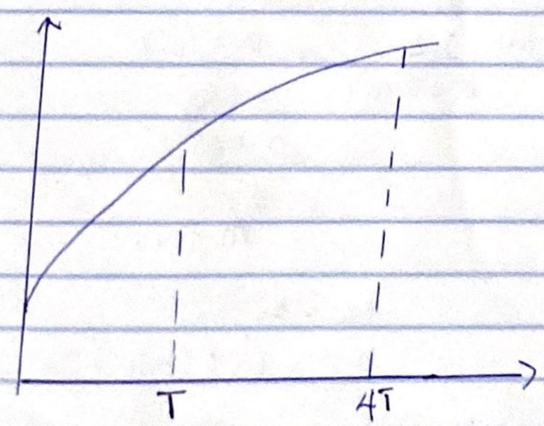
$= 0.981\,Km\,x$

For $t=T$.

$\Delta\% = \left[0.6320-0\right] \times 100\,\% = 63.2\,\%$

$t = 4T$

$\Delta\% = \left[0.981-0\right] \times 100\,\% = 98.1\,\%$



(4) $\frac{\theta_o}{\theta_i}(s) = \frac{1}{3s+1}$

$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$

$\theta_o(s) = \frac{c}{s^2(3s+1)}$

$\theta_o(t) = \frac{c/3}{s^2(s+1/3)}$

$\theta_t = C\left[t - 3C\left[1 - e^{-t/3}\right]\right]$

$\theta_o(t) = C[t - 3(1)]$

$\theta_o(t) = Ct - 3C$

$\theta_t - \theta_2 - \theta_o = Ct - [Ct - 3c] = 3c$

$T = 3$, $c = 4$ mm/s

after 2 seconds.

$\theta = 4 \times 2 = 8$ mm.

$\theta_2 = 4\text{mm} \times 3 = 12$ mm at steady state.

$\theta_o = 4[2 - 3(1 - e^{-2/3}]$

$\theta_o = 2.161$ mm

(5) (i) $\frac{2}{0.2+0.5} = \frac{\frac{2}{0.5}}{\frac{0.2s}{0.5}+1}$ (ii) $\frac{0.2}{0.05s+0.1} = \frac{0.2/0.1}{\frac{0.05s}{0.1}+1}$

$\Rightarrow 4/0.4s+1$ $= 2/0.5s+1$

4 = Dc gain

0.4 = Time constant

2 = Dc gain

0.5 = time constant.

T

5(iii) $2/3s+1$

$2 =$ Dc gain

$3 =$ Time constant

(iv) $\frac{16}{8s+4} = \frac{16/4}{8/4s+1}$

$= \frac{4}{2s+1}$

$4 =$ Dc gain

$2 =$ Time constant

(6) $\frac{\omega}{\theta}(s) = \frac{K_m}{T_m s + 2}$

$K_m = 15 s^{-1}$

$T_m = 4$

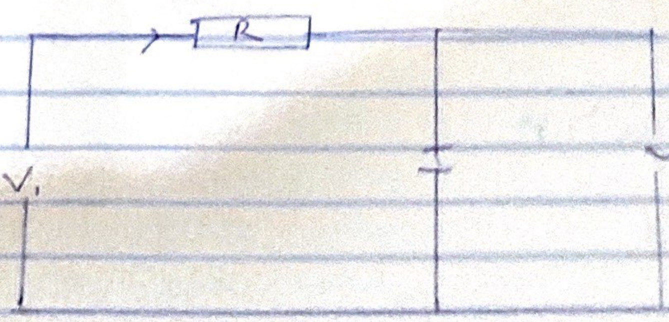
$\frac{15}{4s+2} = \frac{15/2}{\cancel{4s}/2 + 2/2} = \frac{7.5}{2s+1}$

Dc gain $= 7.5\ ms^{-1}$

Time constant $= 2$ sec

SYSTEM RESPONSE 2

(1)



$T = Rc$

$R = 47\ \Omega \quad C = 20\ \mu F$

$Vi = 5\sin(2000t)$

$T = 47 \times 20 \times 10^{-6} = \cancel{8.4\times10^{-4}} = 9.4\times10^{-4}$

$$\left(\frac{V_0}{V_i}\right)s = \frac{1}{Ts+1}$$

$$G(s) = 1/Ts+1$$

$$G(jw) = \frac{1}{9.4\times10^{-9}jw+1} \times \frac{9.4\times10^{-9}jw-1}{9.4\times10^{-9}jw-1}$$

$$G\omega = \frac{9.4\times10^{-9}(jw-1)}{(9.4\times10^{-9})^2 j\omega^2-1}$$

$$G\omega = \frac{-1}{(9.4\times10^{-9})(\omega^2-1)}$$

where $\omega = 2000$ rad/s.

$$\phi = \tan^{-1}\left[\frac{9.4\times10^{-1}(2000)^2-1}{1}\right]$$

$$\phi = -61.99^\circ$$

$$G(jw) = \frac{1}{\sqrt{(9.4\times10^{-9})^2\omega^2+1^2}}$$

$$= \frac{1}{\sqrt{9.4\times10^{-9}(2000)^2+1^2}}$$

$$= 0.4696$$

$$V_0 = 5\times0.4696 = 2.35$$

(2) $$\frac{x_0}{x_i} = \frac{1}{T^2s^2+2\delta Ts+1}$$

$$G(s) = \frac{1}{(1-T^2s^2)+2\delta Ts}$$

$$G(jw) = \frac{1}{[1-T^2w^2]+2\delta Tjw}$$

B R 6

$d = 0.2$, $T = 0.4s$, $W = 2.5$ rad/s.

$G(j\omega) = \dfrac{1 - T^2\omega^2 - 2\delta T j\omega}{(1 - T^2\omega^2) + 4\delta^2 T^2 \omega^2}$

$1 - (0.4)^2 (2.5)^2 - 2(0.2)(0.4)(2.5)$

$[1 - (0.4)^2 (2.5)^2] - 4(0.2)^2 (0.4)(2.5)^2$

$G(j\omega) = \theta = 2.5$

$\phi = \tan^{-1}\left(\frac{2.5}{0}\right)$

$\tan^{-1}(\alpha = 90^\circ$

$|Gj\omega| = \sqrt{0^2 + 2.5^2}$

$|Gj\omega| = 2.5$

Amplitude => 6×2.5

Amplitude => 15