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Math 104 Assignment

1) $y = \sin(6/x^2)$

$y + \Delta y = \sin\left[\frac{6}{(x + \Delta x)^2}\right]$

$\Delta y = \sin\left[\frac{6}{(x + \Delta x)^2}\right] - y$

$\Delta y = \sin\left[\frac{6}{(x + \Delta x)^2}\right] - \sin\left[\frac{6}{x^2}\right]$

Recall: $\sin A - \sin B = 2 \cos\left[\frac{A+B}{2}\right] \sin\left[\frac{A-B}{2}\right]$

$\therefore \Delta y = 2 \cos\left[\frac{12x^2 + 12x\Delta x + 6\Delta x^2}{2x^2(x + \Delta x)^2}\right] \sin\left[\frac{-12x\Delta x - 6\Delta x^2}{2x^2(x + \Delta x)^2}\right]$

$\frac{\Delta y}{\Delta x} = \frac{2 \cos\left[\frac{12x^2 + 12x\Delta x + 6\Delta x^2}{2x^2(x + \Delta x)^2}\right] \sin\left[\frac{-12x\Delta x - 6\Delta x^2}{2x^2(x + \Delta x)^2}\right] \times \frac{6 - 6}{x^3}}{-6\Delta x}$

$\frac{\Delta y}{\Delta x} = \frac{-12 \cos\left[\frac{12x^2 + 12x\Delta x + 6\Delta x^2}{2x^2(x + \Delta x)^2}\right] \cdot \sin\left[\frac{-12x\Delta x - 6\Delta x^2}{2x^2(x + \Delta x)^2}\right]}{x^3}$

$\lim_{\Delta x \rightarrow 0}$

$\lim_{\Delta x \rightarrow 0}$

$6\Delta x$

$\lim_{\Delta x \rightarrow 0}$

$\frac{dy}{dx} = -12 \cos\left[\frac{12x^2}{2x^2(x)^2}\right] \cdot 1$

$\frac{dy}{dx} = \frac{-12 \cos\left[\frac{12x^2}{2x^4}\right]}{x^3} = \frac{-12 \cos\left[\frac{6}{x^2}\right]}{x^3}$

$= \frac{-12 \cos(6/x^2)}{x^3} //$

2) $x = 4t^3 - t^2$

$y = t^4 + 2t^2$

$A = \text{area}; A = \int_a^b y dx$

$$\text{Given } y = t^4 + 2t^2 \text{ then } A = \int_a^b t^4 + 2t^2 dx$$

$$\text{Given } x = 4t^3 - t^2$$

$$\frac{dx}{dt} = 12t^2 - 2t$$

$$dx = (12t^2 - 2t) dt$$

$$\therefore A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t) dt$$

$$A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3) dt$$

$$A = \left[\frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4} \right]_1^3$$

$$A = \left[\frac{12(3)^7}{7} - \frac{2(3)^6}{6} + \frac{24(3)^5}{5} - \frac{4(3)^4}{4} \right] - \left[\frac{12(1)^7}{7} - \frac{2(1)^6}{6} + \frac{24(1)^5}{5} - \frac{4(1)^4}{4} \right]$$

$$A = \frac{160704}{35} - \frac{544}{105}$$

$$= 4586.36 \text{ square units}$$

$$3.) \quad x = 4t^3 - t^2 \quad y = t^4 + 2t^2$$

$$\frac{dx}{dt} = 12t^2 - 2t$$

$$\frac{dy}{dt} = 4t^3 + 4t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{4t^3 + 4t}{12t^2 - 2t}$$

$$= \frac{4t(t^2 + 1)}{2t(6t - 1)} = \frac{2(t^2 + 1)}{6t - 1}$$