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1. Differentiate $y = \sin(6/x^2)$ from the first principle

Solution.

$$y = \sin\left(6/x^2\right)$$

$$y + \Delta y = \sin\left[\frac{6}{(x+\Delta x)^2}\right]$$

$$\Delta y = \sin\left(\frac{6}{(x+\Delta x)^2}\right) - y$$

$$\Delta y = \sin\left(\frac{6}{(x+\Delta x)^2}\right) - \sin\left(\frac{6}{x^2}\right)$$

using the formula $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$,

$$A = \frac{6}{(x+\Delta x)^2}, \quad B = \frac{6}{x^2}$$

$$\Delta y = 2\cos\frac{\left(\frac{6}{(x+\Delta x)^2} + \frac{6}{x^2}\right)}{2}\sin\frac{\left(\frac{6}{(x+\Delta x)^2} - \frac{6}{x^2}\right)}{2}$$

$$= 2\cos\frac{\left(\frac{6x^2 + 6(x+\Delta x)^2}{x^2(x+\Delta x)^2}\right)}{2}\sin\frac{\left(\frac{6x^2 - 6(x+\Delta x)^2}{x^2(x+\Delta x)^2}\right)}{2}$$

$$= 2\cos\left(\frac{6x^2 + 6x^2 + 12x\Delta x + 6(\Delta x)^2}{2x^2(x+\Delta x)^2}\right)\sin\left(\frac{6x^2 - 6x^2 - 12x\Delta x - 6(\Delta x)^2}{2x^2(x+\Delta x)^2}\right)$$

$$\Delta y = 2\cos\left(\frac{12x^2 + 12x\Delta x + 6(\Delta x)^2}{2x^2(x+\Delta x)^2}\right)\sin\left(\frac{-12x\Delta x - 6(\Delta x)^2}{2x^2(x+\Delta x)^2}\right)$$

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\left(\frac{12x^2 + 12x\Delta x + 6(\Delta x)^2}{2x^2(x+\Delta x)^2}\right)\sin\left(\frac{-12x\Delta x - 6(\Delta x)^2}{2x^2(x+\Delta x)^2}\right)}{\Delta x}$$

$$= \frac{2\cos\left[\frac{2\left(6x^2 + 6x\Delta x + 3(\Delta x)^2\right)}{2\left[x^2(x+\Delta x)^2\right]}\right]\sin\left(\frac{2\left(-6x\Delta x - 3(\Delta x)^2\right)}{2\left[x^2(x+\Delta x)^2\right]}\right)}{\Delta x}$$

$$= \frac{2\cos\left(\frac{6x^2 + 6x\Delta x + 3(\Delta x)^2}{x^2(x+\Delta x)^2}\right)\sin\left(\frac{-6x\Delta x - 3(\Delta x)^2}{x^2(x+\Delta x)^2}\right)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = 2\cos\left(\frac{6x^2+6x\Delta x+3(\Delta x)^2}{x^2(x+\Delta x)^2}\right)\frac{\sin\left(\dfrac{-6x\Delta x-3(\Delta x)^2}{x^2(x+\Delta x)^2}\right)}{\Delta x}$$

$$= 2\cos\left(\frac{6x^2+6x\Delta x+3(\Delta x)^2}{x^2(x+\Delta x)^2}\right)\frac{\sin\left[\Delta x\left(\dfrac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2\Delta x}\right)\right]}{\Delta x}$$

$$= 2\cos\left(\frac{6x^2+6x\Delta x+3(\Delta x)^2}{x^2(x+\Delta x)^2}\right)\frac{\sin\left[\Delta x\left(\dfrac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2\Delta x}\right)\right]\left(\dfrac{-6x-3\Delta x}{x^2(x+\Delta x)^2}\right)}{\Delta x\cdot\left(\dfrac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2\Delta x}\right)}$$

$$* \lim_{\Delta x\to 0}\left\{\frac{\sin\left[\Delta x\left(\dfrac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2\Delta x}\right)\right]}{\Delta x\left(\dfrac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2\Delta x}\right)}\right\} = 1$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0} 2\left[\cos\left(\frac{6x^2+6x\Delta x+3(\Delta x)^2}{x^2(x+\Delta x)^2}\right)\right]\cdot\lim_{\Delta x\to 0}\left(\frac{-6x-3\Delta x}{x^2(x+\Delta x)^2}\right)$$

* [illegible]

$$= \lim_{\Delta x\to 0}\left[\cos\left(\frac{6x^2+6x\Delta x+3(\Delta x)^2}{x^4+2x^3\Delta x+x^2\Delta x}\right)\right]\cdot\lim_{\Delta x\to 0} 2\left(\frac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2}\right.$$

$$\frac{dy}{dx} = \cos\left(\frac{6x^2}{x^4}\right)\cdot 2\left(-\frac{6x}{x^4}\right)$$

$$= \cos\left(\frac{6}{x^2}\right)\cdot\frac{-12x}{x^4}$$

$$\frac{dy}{dx} = \frac{-12}{x^3}\cos\left(\frac{6}{x^2}\right)$$

$$\frac{dy}{dx} = -12x^{-3}\cos\left(6x^{-2}\right)$$

2 Find the area under the curve, given parametric equations $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$ at $t = 1$ and $t = 3$.

Solution.

$x = 4t^3 - t^2 \qquad y = t^4 + 2t^2, \quad a = 1, \quad b = 3$

$A = \int_a^b y\,dx$

$= \int_1^3 (t^4 + 2t^2)\,dx$

$x = 4t^3 - t^2$

$\frac{dx}{dt} = 12t^2 - 2t$

$dx = (12t^2 - 2t)\,dt$

$A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)\,dt$

$= \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3)\,dt$

$= \left[\frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4}\right]_1^3$

$= \left[\frac{12}{7}t^7 - \frac{t^6}{3} + \frac{24}{5}t^5 - t^4\right]_1^3$

$= \left[\frac{12}{7}(3)^7 - \frac{(3)^6}{3} + \frac{24}{5}(3)^5 - (3)^4\right] - \left[\frac{12}{7}(1)^7 - \frac{(1)^6}{3} + \frac{24}{5}(1)^5 - (1)^4\right]$

$= \frac{160704}{35} - \frac{544}{105}$

$\cancel{A = \frac{67096}{35} = 1917.03 \text{ sq. units}}$

$A = 4586.36$ sq. units.

3 If $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$, find dy/dx.

Solution.

$x = 4t^3 - t^2, \quad y = t^4 + 2t^2$

$\frac{dx}{dt} = 12t^2 - 2t \qquad \frac{dy}{dt} = 4t^3 + 4t$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$= 4t^3 + 4t \div 12t^2 - 2t$$

$$= \frac{4t^3 + 4t}{12t^2 - 2t}$$

$$\cancel{\frac{dy}{dt} = \frac{t^3 + t}{3t^2 - \frac{1}{2}t} = \frac{2t^3 + 2t}{6t^2 - t}}$$

$$= \frac{2t(2t^2 + 2)}{2t(6t - 1)}$$

$$\frac{dy}{dx} = \frac{2t^2 + 2}{6t - 1}$$