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DEPARTMENT: ELECTRICAL/ELECTRONICS ENGINEERING

COURSE CODE/TITLE: MAT 102 → VECTORS, GEOMETRY ~~AND DYNAMICS~~

MATRIC NO: 19/ENG04/047

14-5-20 SERIAL NO: 16

1. Find a unit tangent to the space curve $x = t$, $y = t^2$, $z = t^3$ at the point where $t = 1$.

Solution

$x = t$, $y = t^2$, $z = t^3$

position vector, $\vec{r} = x\underline{i} + y\underline{j} + z\underline{k}$

$$\vec{r} = (t)\underline{i} + (t^2)\underline{j} + (t^3)\underline{k}$$

$$\frac{d\vec{r}}{dt} = \underline{i} + (2t)\underline{j} + (3t^2)\underline{k}$$

At point $t = 1$,

$$\frac{d\vec{r}}{dt} = \underline{i} + 2(1)\underline{j} + 3(1)^2\underline{k}$$

$$\frac{d\vec{r}}{dt} = \underline{i} + 2\underline{j} + 3\underline{k}$$

$$\left|\frac{d\vec{r}}{dt}\right| = \sqrt{(1) + (2) + (3)}$$

$$= \sqrt{1 + 4 + 9}$$

$$\left|\frac{d\vec{r}}{dt}\right| = \sqrt{14}$$

$$T = \frac{d\vec{r}/dt}{|d\vec{r}/dt|}$$

$$T = \frac{\underline{i} + 2\underline{j} + 3\underline{k}}{\sqrt{14}}$$

2. Given that $A = (4t^3)\underline{j} + 5\underline{k}$

$B = (2t^2)\underline{i} + (4t)\underline{j}$

$G = A \times B$

Find the integral of G from (0 to 1).

Solution

$$G = A \times B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4t^3 & 5 \\ 2t^2 & 4t & 0 \end{vmatrix}$$

$$= \underline{i}\begin{vmatrix} 4t^3 & 5 \\ 4t & 0 \end{vmatrix} - \underline{j}\begin{vmatrix} 0 & 5 \\ 2t^2 & 0 \end{vmatrix} + \underline{k}\begin{vmatrix} 0 & 4t^3 \\ 2t^2 & 4t \end{vmatrix}$$

$$= \underline{i}\left[(4t^3 \times 0) - (5 \times 4t)\right] - \underline{j}\left[(0 \times 0) - (5 \times 2t^2)\right] + \underline{k}\left[(0 \times 4t) - (2t^2 \times 4t^3)\right]$$

$$= \underline{i}\left[0 - 20t\right] - \underline{j}\left[0 - 10t^2\right] + \underline{k}\left[0 - 8t^5\right]$$

$$G = -(20t)\underline{i} + (10t^2)\underline{j} - (8t^5)\underline{k}$$

$$\int_0^1 G = \left\{\left[\int -20t\right]\underline{i} + \left[\int 10t^2\right]\underline{j} + \left[\int -8t^5\right]\underline{k}\right\}_0^1$$

$$= \left[\frac{-20t^2}{2}\underline{i} + \frac{10t^3}{3}\underline{j} - \frac{8t^6}{6}\underline{k}\right]_0^1$$

$$\cancel{\int_0^1 G = \left[(-10t^2)\underline{i} + (5t^2)\underline{j} - \left(\tfrac{8}{5}t^5\right)\underline{k}\right]_0^1}$$

$$\cancel{= \left[-10(1)^2\underline{i}\right] + \left[5(1)^2\underline{j}\right] - \left[\tfrac{8}{5}(1)^3\underline{k}\right] - [0]}$$

$$\cancel{\int_0^1 G = -10\underline{i} + 5\underline{j} - \tfrac{8}{5}\underline{k}}$$

$$\int_0^1 G = \left[(-10t^2)\underline{i} + \left(\tfrac{10}{3}t^3\right)\underline{j} - \left(\tfrac{4}{3}t^6\right)\underline{k}\right]_0^1$$

$$= \left[-10(1)^2\underline{i}\right] + \left[\tfrac{10}{3}(1)^3\underline{j}\right] - \left[\tfrac{4}{3}(1)^6\underline{k}\right]$$

$$\int_0^1 G = -10\underline{i} + \frac{10}{3}\underline{j} - \frac{4}{3}\underline{k}$$