

# MODELLING OF THE OPERATION OF A PERMANENT MAGNET SYNCHRONOUS MACHINE

## Abstract

A permanent magnet synchronous machine is one of the best and widely used methods for machine control applications. The PMSM is widely used in robotics, machine tools, actuators, and it is being considered in high-power applications. The PMSM is known for having low torque ripple, superior dynamic performance, high efficiency and high power density. This paper is going to be dealing with the introduction of PMSM and how it is evolved from synchronous motors, I'll also briefly discuss about the different types of PMSM, The modeling of PMSM and it derives the equivalent circuit of PMSM. Also permanent magnet synchronous motor drive system is briefly discussed with explanation of each block in the systems. The paper is also going to talk about various control techniques of PMSM like the scalar control and vector control.

## Introduction

A synchronous electric motor is an AC motor in which, at steady state, the rotation of the shaft is synchronized with the frequency of the supply current; the rotation period is exactly equal to an integral number of AC cycles. Synchronous motors contain multiphase AC electromagnets on the stator of the motor that create a magnetic field which rotates in time with the oscillations of the line current. The rotor with permanent magnets or electromagnets turns in step with the stator field at the same rate and as a result, provides the second synchronized rotating magnet field of any AC motor. A synchronous motor is termed doubly fed if it is supplied with independently excited multiphase AC electromagnets on both the rotor and stator.

The synchronous motor and induction motor are the most widely used types of AC motor. The difference between the two types is that the synchronous motor rotates at a rate locked to the line frequency since it does not rely on current induction to produce the rotor's magnetic field. By contrast, the induction motor requires slip: the rotor must rotate slightly slower than the AC alternations in order to induce current in the rotor winding. Small synchronous motors are used in timing applications such as in synchronous clocks, timers in appliances, tape recorders and precision servomechanisms in which the motor must operate at a

precise speed; speed accuracy is that of the power line frequency, which is carefully controlled in large interconnected grid systems.

Synchronous motors are available in self-excited sub-fractional horsepower sizes to high power industrial sizes. In the fractional horsepower range, most synchronous motors are used where precise constant speed is required. These machines are commonly used in analog electric clocks, timers and other devices where correct time is required. In higher power industrial sizes, the synchronous motor provides two important functions. First, it is a highly efficient means of converting AC energy to work. Second, it can operate at leading or unity power factor and thereby provide power-factor correction.

One of the types of synchronous motor is the PMSM. The PMSM consists of conventional three phase windings in the stator and permanent magnets in the rotor. The purpose of the field windings in the conventional synchronous machine is done by permanent magnets in PMSM. The conventional synchronous machine requires AC and DC supply, whereas the PMSM requires only AC supply for its operation. One of the greatest advantages of PMSM over its counterpart is the removal of dc supply for field excitation.

The development of PMSM has happened due to the invention of novel magnetic materials and rare earth materials. PMSM give numerous advantages in scheming recent motion management systems. Energy efficient PMSM are designed due to the availability of permanent magnet materials of high magnetic flux density.

In synchronous motors the rotor rotates at the speed of stator revolving field. The speed of the revolving stator field is called as synchronous speed. The synchronous speed ( $\omega_s$ ) can be found by the frequency of the stator input supply ( $f_s$ ), and the number of stator pole pairs ( $p$ ). The stator of a three phase synchronous motor consists of distributed sine three phase winding, whereas the rotor consists of the same number of  $p$ -pole pairs as stator, excited by permanent magnets or a separate DC supply source.

When the synchronous machine is excited with a three phase AC supply, a magnetic field rotates at synchronous speed develops in the stator. The synchronous speed of this rotating magnetic field is shown.

$$N=120f_s/P_{rpm} \text{ Eq1}$$

Where  $N$ , synchronous speed

$f_s$ , frequency of AC supply in Hz;

$P$ , number of poles;

$p$ , pole pairs and it is given by  $(P/2)$ .

## Types of PMSM

The PMSM are classified based on the direction of field flux are as follows,

1. Radial field
2. Axial field

In radial field, the flux direction is along the radius of the machine. The radial field permanent magnet motors are the most commonly used. In axial field, the flux direction is parallel to the rotor shaft. The axial field permanent magnet motors are presently used in a variety of numerous applications because of their higher power density and quick acceleration.

The permanent magnets can be placed in many different ways on the rotor of PMSM. This type of arrangement provides the highest air gap flux density, but it has the drawback of lower structural integrity and mechanical robustness. Machines with this arrangement of magnets are known as Surface mount PMSMs. The development of this arrangement is more difficult than the surface mount or inset magnet permanent magnet rotors. The inset permanent magnet rotor construction has the advantages of both the surface and interior permanent magnet rotor arrangements by easier construction and mechanical robustness, with a high ratio between the quadrature and direct-axis inductances, respectively.

The surface PMSM with radial flux are generally applied for applications which require low speed operations. These machines have the advantage of high power density than the other types of PMSM. The interior PMSM are used for applications which require high speed.

The principle of operation is identical for all the types of PMSM, in spite of the types of mounting the permanent magnets in the rotor.

The important significance of the type of mounting the permanent magnets on the rotor is the variation in direct axes and quadrature axes inductance values, which is explained below. The primary path of the flux through the permanent magnets rotor is the direct axis. The stator inductance when measured in the position of permanent magnets aligned with stator winding is called as direct axis inductance. The quadrature axis inductance is measured by rotating the magnets from the already aligned position (direct axis) by  $90^\circ$ , in this position the iron (inter polar area of the rotor) sees the stator flux. The flux density of the permanent magnet materials is presently high and its permeability is almost equal to that of the air, such that the air gap between the rotor and stator of PMSM can be treated as an extension of permanent magnet thickness. The reluctance of direct axis is always greater than the quadrature axis reluctance, since the effectual air gap of the direct axis is several times that of the real air gap looked by the quadrature axis.

The significance of such an uneven reluctance is that the direct axis inductance is greater than the quadrature axis inductance and it is shown in Eq. (2).

$$L_d > L_q \quad \text{Eq(2)}$$

Where  $L_d$  is the inductance along the direct to the magnet axis and  $L_q$  is the inductance along the axis in quadrature to the magnet axis.

### **3. Modeling of PMSM**

For proper simulation and analysis of the system, a complete modelling of the drive model is essential. The motor axis has been developed using d-q rotor reference frame theory. At any particular time  $t$ , the rotor reference axis makes an angle  $\theta_r$  with the fixed stator axis and the rotating stator mmf creates an angle  $\alpha$  with the rotor d axis. It is viewed that at any time  $t$ , the stator mmf rotates at the same speed as that of the rotor axis.

The required assumptions are obtained for the modelling of the PMSM without damper windings.

1. Saturation is neglected.
2. Induced EMF is sinusoidal in nature.
3. Hysteresis losses and Eddy current losses are negligible.
4. No field current dynamics.

Voltage equations from the model are given by,

$$V_q = R_s i_q + \omega_r \lambda_d + \rho \lambda_q \quad \text{Eq3}$$

$$V_d = R_s i_d - \omega_r \lambda_q + \rho \lambda_d \quad \text{Eq4}$$

Flux linkages are given by,

$$\lambda_q = L_q i_q \quad \text{Eq5}$$

$$\lambda_q = L_q i_q + \lambda_f \quad \text{Eq6}$$

Substituting Eq. (5) and Eq. (6) into Eq. (3) and Eq. (4)

$$V_q = R_s i_q + \omega_r L_d i_d + \lambda_f + \rho L_d i_d \quad \text{Eq7}$$

$$V_d = R_s i_d - \omega_r L_q i_q + \rho L_d i_d + \lambda_f \quad \text{Eq8}$$

Arranging Eq. (7) and Eq. (8) in matrix form,

$$\begin{bmatrix} V_q \\ V_d \end{bmatrix} = \begin{bmatrix} R_s & \rho L_d \\ \rho L_q & R_s \end{bmatrix} \begin{bmatrix} i_q \\ i_d \end{bmatrix} + \begin{bmatrix} \omega_r L_d \\ -\omega_r L_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \lambda_f \\ \lambda_f \end{bmatrix} \quad \text{Eq9}$$

The developed torque motor is being given by,

$$T_e = 3/2 P (\lambda_d i_q - \lambda_q i_d) \quad \text{Eq10}$$

The mechanical torque equation is,

$$T_e = T_L + B \omega_m + J d\omega_m/dt \quad \text{Eq11}$$

Solving for the rotor mechanical speed from Eq. (11)

$$\omega_m = \int (T_e - T_L - B \omega_m) / J dt \quad \text{Eq12}$$

and

$$\omega_m = \omega_r / 2P \quad \text{Eq13}$$

In the above equations  $\omega_r$  is the rotor electrical speed,  $\omega_m$  is the rotor mechanical speed.

## Parks transformation and dynamic d-q modeling

The dynamic d-q modelling of the system is used for the study of motor during transient state and as well as in the steady state conditions. It is achieved by converting the three phase voltages and currents to dqo axis variables by using the Parks transformation.

Converting the phase voltages variables  $V_{abc}$  to  $V_{dqo}$  variables in rotor reference frame axis are illustrated in the equations,

$$\begin{bmatrix} V_d \\ V_q \\ V_o \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta_r & \sin\theta_r & 1 \\ -\frac{1}{2}\cos\theta_r - \frac{\sqrt{3}}{2}\sin\theta_r & \frac{1}{2}\cos\theta_r - \frac{\sqrt{3}}{2}\sin\theta_r & 1 \\ \frac{1}{2}\cos\theta_r + \frac{\sqrt{3}}{2}\sin\theta_r & \frac{1}{2}\cos\theta_r + \frac{\sqrt{3}}{2}\sin\theta_r & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \text{Eq14}$$

Convert  $V_{dqo}$  to  $V_{abc}$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta_r & \sin\theta_r & 1 \\ -\cos\theta_r & \sin\theta_r & 1 \\ \cos\theta_r & -\sin\theta_r & 1 \end{bmatrix} \begin{bmatrix} V_d \\ V_q \\ V_o \end{bmatrix} \quad \text{Eq15}$$

## Equivalent circuit of PMSM

Equivalent circuit is essential for the proper simulation and designing of the motor. It is achieved and derived from the d-q modelling of the motor using the voltage equations of the stator. From the assumption, rotor d axis flux is represented by a constant current source which is described through the following equation,

$$\lambda_f = L_{dm} i_f \quad \text{Eq16}$$

where  $\lambda_f$ , field flux linkage;  $L_{dm}$ , d-axis magnetizing inductance;  $i_f$ , equivalent permanent magnet field current.

Permanent magnet synchronous motor drive system

The motor drive essentially consists of four main components such as the PMSM, the inverter, the main control unit and the position sensor.

## **Inverter**

For variable frequency and magnitude, voltage source inverters are devices which convert the constant DC voltage level to variable AC voltage. As specified in the function, these inverters are commonly used in adjustable speed drives.

Three phase inverters consist of a DC voltage source and six power ON/OFF switches connected to the PMSM. Selection of the inverter switches must be carefully done based on the necessities of operation, ratings and the application. There are several devices available in the market and these are thyristors, bipolar junction transistors (BJTs), MOS field effect transistors (MOSFETs), insulated gate bipolar transistors (IGBTs) and gate turn off thyristors (GTOs). It has been inferred that MOSFETs and IGBTs are preferred in the industry because of its advantages that the MOS gating permits high power gain and control advantages. MOSFET is considered to be universal power ON/OFF device for low power and low voltage applications, whereas IGBT has wide acceptance in the motor drive applications and other application in the low and medium power range. The power devices when used in motor drives applications require an inductive motor current path provided by antiparallel diodes when the switch is turned off.

## **5. Control techniques of PMSM**

Many techniques based on both motor designs and control techniques that have been proposed in literature to diminish the torque ripples in the PMSM.

### **Classification of the various control techniques.**

#### **Scalar control**

One way of controlling AC motors for variable speed applications is through the open loop scalar control, which represents the most popular control strategy of squirrel cage AC motors. It is presently used in applications where information about the angular speed need not be known. It is suitable for a wide range of drives as it ensures robustness at the cost of reduced dynamic performance. Typical applications are pump and fan drives and low-cost drives. The main idea of this method is the variation of the supply voltage frequency inattentively from the shaft response (position, angular speed). The magnitude of the supply voltage is changed according to the frequency in a constant ratio. The motor is then in the

condition where the magnetic flux represents the nominal value and the motor is neither over excited nor under excited. The major advantage of this simple method is running in a sensorless mode because the control algorithm does not need information about the angular speed or actual rotor position. On the contrary, the significant disadvantages are the speed dependence on the external load torque, mainly for PMSM, and the reduced dynamic performances.

### **Vector control**

The vector control of PMSM allows separate closed loop control of both the flux and torque, thereby achieving a similar control structure to that of a separately excited DC machine, as discussed.

### **Direct torque control (DTC)**

The DTC is one of the high performance control strategies for the control of AC machine. In a DTC drive applications, flux linkage and electromagnetic torque are controlled directly and independently by the selection of optimum inverter switching modes of operation. To acquire a faster torque output, low inverter switching frequency and low harmonic losses in the model, the selection is made to restrict the flux linkages and electromagnetic torque errors within the respective flux and torque hysteresis bands. The required optimal switching vectors can be selected by using the optimum switching voltage vector look-up table. This can be obtained by simple physical considerations involving the position of the stator-flux linkage space vector, the available switching vectors, and the required torque flux linkage.

### **Field oriented control (FOC) of PMSM**

For the control of PM motors, FOC technique is used for synchronous motor to evaluate as a DC motor. The stator windings of the motor are fed by an inverter that generates a variable frequency variable voltage scheme. Instead of controlling the inverter frequency independently, the frequency and phase of the output wave are controlled using a position sensor.

FOC was invented in the beginning of 1970s and it demonstrates that an induction motor or synchronous motor could be controlled like a separately excited DC motor by the orientation of the stator mmf or current vector in relation to the



rotor flux to achieve a desired objective. For the motor to behave like a DC motor, the control needs knowledge of the position of the instantaneous rotor flux or rotor position of permanent magnet motor. This needs a resolver or an absolute optical encoder. Knowing the position, the three phase currents can be calculated. Its calculation using the current matrix depends on the control desired. Some control options are constant torque and flux weakening. These options are based in the physical limitation of the motor and the inverter. The limit is established by the rated speed of the motor, at which speed the constant torque operation finishes and the flux weakening starts. From the literature it has been found that the best control for PMSM to make it to behave like a DC motor using decoupling control is known as vector control or field oriented control. The torque components of flux and currents in the motor are separated by the vector control through its stator excitation.

Steady state torque versus speed.

From the dynamic model of the PMSM, the vector control is derived.

Assuming the line currents as input signals,

$$i_a = I_m \sin(\omega_r t + \alpha) \quad \text{Eq17}$$

$$i_b = I_m \sin(\omega_r t + \alpha - 2\pi/3) \quad \text{Eq18}$$

$$i_c = I_m \sin(\omega_r t + \alpha + 2\pi/3) \quad \text{Eq19}$$

Writing the above Eq. (17) to Eq. (19) in the matrix form,

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} \cos(\omega_r t + \alpha) \\ \cos(\omega_r t + \alpha - 2\pi/3) \\ \cos(\omega_r t + \alpha + 2\pi/3) \end{bmatrix} I_m \quad \text{Eq20}$$

where  $\alpha$  is the angle between the rotor field and stator current phasor,  $\omega_r$  is the electrical rotor speed.

Using the Park's transformation, the currents obtained in the previous cycle are transformed to the rotor reference frame axis with the rotor speed  $\omega_r$ . Since  $\alpha$  is fixed for a given load torque, the q and d axis currents are fixed in the rotor reference frames. These constant values are made similar to the armature and field currents in the separately excited DC machine. The q axis current is distinctly equivalent to the armature current of the DC machine. The d axis current is field current, but not in its entirety. It is only a partial field current; the other part is

contributed by the equivalent current source representing the permanent magnet field. Thus, the q axis current is known as the torque producing component and the d axis current is called the flux producing component of the stator currents.

Substituting Eq. (20) in Eq. (14) and obtaining  $i_d$  and  $i_q$  in terms of  $I_m$  as follows,

$$i_d = I_m \sin \alpha \cos \alpha \quad \text{Eq 21}$$

Using Eq. (3), Eq. (4), Eq. (10) and Eq. (21) the electromagnetic torque equation is obtained as given below,

$$T_e = \frac{3}{2} P \frac{2}{\lambda} L_d - L_q I_m^2 \sin 2\alpha + \lambda f I_m \sin \alpha \quad \text{Eq 22}$$

Where  $L_d$  and  $L_q$  are the d and q axis synchronous inductances. Each of the two terms in the equation has a useful physical interpretation. The first “magnet” torque term is independent of  $i_d$  but is directly proportional to the stator current component  $i_q$ . In contrast, the second reluctance torque term is proportional to the  $i_d$  and  $i_q$  current component product and to the difference of the inductance values.

As Eq. (22) shows that the torque depends on the rotor type and its inductances  $L_d$ ,  $L_q$  and on permanent magnets mounted on the rotor. The non-salient PMSM have surface mounted magnets on the rotor and the reluctance term disappears since  $L_q$  equals  $L_d$ . On the contrary, the electromagnetic torque is more dominated by the reluctance component when permanent magnets are interior mounted and the rotor's saliency causes a difference in  $L_q$  and  $L_d$ .