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COVID-19 HOLIDAY ASSIGNMENT

Question 1

Find the point of intersection of the following line on the circle
$x - y - 14 = 0$ and $x^2 + y^2 - 6x + 8y = 0$

SOLUTION

Equation of line: $y = x - 14$ ……(i)

Equation of circle: $x^2 + y^2 - 6x + 8y = 0$ …….(ii)

Substituting equation (i) in equation (ii) gives;

$= x^2 + (x - 14)^2 - 6x + 8(x - 14) = 0$

$= x^2 + (x - 14)(x - 14) - 6x + 8x - 112 = 0$

$= x^2 + x^2 - 14x - 14x + 196 - 6x + 8x - 112 = 2x^2 - 28x + 196 + 2x - 112 = 0$

$= 2x^2 - 26x + 84 = 0$

$= x^2 - 13x + 42 = 0$

$(x - 6)(x - 7) = 0$

$x = 7, 6$

Substituting the values of x in equation (i) gives;

$y = x - 14$

$y = -7, -8$

∴ The points of intersection are $(7, -7)$ and $(6, -8)$

Question 2

Find the point of intersection of the following line on the circle
$2x + y - 10 = 0$ and $x^2 + y^2 + 4x - 6y = 0$

SOLUTION

Equation of line: $y = -2x + 10$ …….(i)

Equation of circle: $x^2 + y^2 + 4x - 6y = 0$ ……..(ii)

Substituting equation (i) in equation (ii) gives;

$= x^2 + (-2x + 10)^2 + 4x - 6(-2x + 10) = 0$

$= x^2 + (-2x + 10)(-2x + 10) - 6x + 8x - 112 = 0$

$= x^2 + 4x^2 - 20x - 20x + 100 + 4x + 12x - 60 = 0$

$= 5x^2 - 40x + 100 + 16x - 60 = 0$

$= 5x^2 - 24x + 40 = 0$

Using formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(5)(40)}}{2(5)}$

$x = \frac{24 \pm \sqrt{576-800}}{2(5)}$

$x = \frac{24 \pm \sqrt{-224}}{2(5)}$

It cannot factorize fully.

The discriminant $b^2 - 4ac = -224$, which is a negative. This means that there are no real roots.

∴ The line and circle do not intersect. The line $y = \mathbf{-2x + 10}$ misses the circle.

Question 3

Find the point of intersection of the following line on the circle
$x - 5y - 2 = 0$ and $x^2 + 25y^2 - 6xy - 16 = 0$

SOLUTION

Equation of line: $x - 5y - 2 = 0$

$5y = x - 2$

$y = \frac{x - 2}{5}$ …….(i)

Equation of circle: $x^2 + 25y^2 - 6xy - 16 = 0$ ……..(ii)

$= x^2 + 25\left(\frac{x - 2}{5}\right)^2 - 6x(\frac{x - 2}{5}) - 16 = 0$

$= x^2 + 25\left(\frac{x - 2}{5}\right)\left(\frac{x - 2}{5}\right) - \left(\frac{6x^2 - 12x}{5}\right) - 16 = 0$

$= x^2 + 25\left(\frac{x^2 - 4x + 4}{25}\right) - \left(\frac{6x^2 - 12x}{5}\right) - 16 = 0$

$= x^2 + x^2 - 4x + 4 - \left(\frac{6x^2 - 12x}{5}\right) - 16 = 0$

$= x^2 + x^2 - 4x + 4 - \frac{6x^2}{5} + \frac{12x}{5} - 16 = 0$

$= 2x^2 - \frac{6x^2}{5} - 4x + \frac{12x}{5} + 4 - 16 = 0$

$= \frac{10x^2 - 6x^2}{5} - \frac{20x + 12x}{5} - 12 = 0$

$= \frac{4x^2}{5} - \frac{32x}{5} - 12 = 0 \times 5$

$= 4x^2 - 32x - 60 = 0 \div 4$

$= x^2 - 8x - 15 = 0$

$(x - 3)(x - 5) = 0$

$x = 3, 5$

Substituting the values of x in equation (i) gives;

$y = \frac{x - 2}{5}$

$y = \frac{1}{5}, \frac{3}{5}$

∴ The points of intersection are $(3, \mathbf{\frac{1}{5}})$ and $(5, \mathbf{\frac{3}{5}})$