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Modeling and Simulation of Permanent Magnet Synchronous Motor Drives

ABSTRACT

This paper addresses the modeling and simulation of permanent magnet synchronous motor (PMSM) supplied from a six-step continuous inverter based on state-space method. Two transformations of reference frame are used and the method on deciding the initial conditions is presented. The results obtained show that the method is very effective.

Keywords-Simulation, PMSM, model, inverter.

INTRODUCTION

Permanent magnet synchronous motor (PMSM) is a class of synchronous motor that high magnet material is used to magnetize, it has characteristics of high efficiency, simple structure, easy to control and so on. Inverter fed PMSM drive systems have made it come true to eliminate the physical contact between the mechanical brushes and commutators, so it have become a viable choice for motion control applications such as robotics, numerically controlled machine tools, electric propulsion, aerospace, and many more. With the fast development of power electronic and decreasing price of electrical parts and apparatus, inverter fed PMSM have been widely used in industry. In view of the growing use of these systems, it is important to have the ability to simulate the inverter fed PMSM drive system so that its characteristics under steady state, transient and abnormal conditions can be predicted. At present, there are many papers to study the starting behavior of it, but few researchers pay attention to the steady state characteristics using simulation method. P. C. Krause [3] studied the steady-state behavior of PMSM by using harmonic analysis method. The author had to calculate the harmonic coefficients in the Fourier series expression of the input voltages, and it may be a very troublesome work. In the course of simulation, some harmonic components had to be neglected, and resulted in inaccurate results. With the development of control theory and application of computer, the state space theory has been proven to be an effective tools to analyze the behavior of the ac electrical machine. Based on state space modeling of the ac motor, the governing differential equations of the motor can be analyzed and solved directly in time domain, and it is very effective to analyze the ac electrical machine supplied fiom inverter, But stead-state analysis is more complex than transient analysis using state space method. The more difficult task is to decide the initial conditions. On deciding the initial conditions, recursive method is used for conventional methods after defining the initial values. It may spend amount of time and there is a problem of convergence. The Ref. [2] studied the method to choose the initial conditions for ac motors, but the method is unfit for PMSM drives. This paper presents a new method to decide the initial conditions, it can improve the simulation results by using the method and can save much time comparing with the conventional method.

VOLTAGE EQUATIONS IN STATIONARY REFERENCE FRAME

A 2-pole, 3-phase, wye-connected, salient-pole PMSM is discussed in the paper, The 3-phase stator windings are identical sinusoidal distributed windings displaced 120" , with N_s equivalent turns and resistance Rand the rotor is a permanent magnet one. The follow assumptions are made in the derivation:

1. Saturation is neglected.
2. The induced EMF is sinusoidal.
3. There is no cage on the rotor.

With these assumptions, the governing differential equations describing the dynamic behavior of PMSM may be written as:

$$\overline{V}_{abcs}=\overline{R}\,\overline{l}abcs + p\overline{\lambda}_{abcs}$$

Where

$$\overline{f}_{abcs}=[f_{as}\ f_{bs}\ f_{cs}]^T$$

$$\overline{R}=diag[\mathrm{R\ R\ R}]$$

The flux linkages λ_{abcs} may be written

$$\overline{\lambda}_{abcs} = L_s \bar{I} + \overline{\lambda}_m$$

Where the inductance matrix, L_s is a 3-by-3, symmetric, positive definite matrix whose elements can be defined by

$$L_{kk} = L_a - L_b \cos(2\theta_r + \frac{2(k-1)x}{3}) \quad k=1,2,3$$

$$L_{12} = L_{21} = -\frac{1}{2}L_a - L_b \cos(2\theta_r - \frac{2\pi}{3})$$

$$L_{13} = L_{31} = -\frac{1}{2}L_a - L_b \cos(2\theta_r + \frac{2\pi}{3})$$

$$L_{23} = L_{32} = -\frac{1}{2}L_a - L_b \cos(2\theta_r)$$

L_a is the average value of the winding inductance

L_b represents the amplitude of variation in the inductance due to the non-uniformity of the air gap

$$\overline{\lambda}_m = K_e \begin{bmatrix} \sin\theta r \\ \sin(\theta r - \frac{2\pi}{3}) \\ \sin(\theta r + \frac{2\pi}{3}) \end{bmatrix}$$

Where K_e , is the electromotive force constant, θ_r,is the angular position of the rotor which is a function of time and it is given by

$$\theta_r = \int_0^t \overline{w}_r(\xi) d\,\xi + \theta_r(0)$$

In the eqution above, ξ is a dummy variable of integration and $\theta_r(0)$ is the time zero position of θ_r, which is generally selected to be zero.

VOLTAGE AND TORQUEE QUATIONS IN THE ROTOR REFERENCE FRAMES

The voltage equations in machine variables has a time-varying inductance matrix, and it is difficult to analyze and simulate. By using Park's transformation, the problem can be solved. The Park's transformation matrix, K_s^r are given by

$$K_s^r = \frac{2}{3}\begin{bmatrix} \cos\theta r & \cos(\theta r - \frac{2\pi}{3}) & \cos(\theta r + \frac{2\pi}{3}) \\ \sin\theta r & \sin(\theta r - \frac{2\pi}{3}) & \sin(\theta r + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

The q, d variables are defined by $f_{qd0}^r = K_s^r f_{abcs}$

From the Park's transformation we can get the voltage equations in the rotor reference frame

$$v_q^r = R i_q^r + \overline{w}_r \lambda_d^r + p\lambda_q^r$$

$$v_d^r = R i_d^r - \overline{w}_r \lambda_q^r + p\lambda_d^r$$

Where

$$\lambda_q^r = L_q\, i_q^r$$

$$\lambda_d^r = L_d\, i_d^r + K_e$$

And

$$L_q = \frac{3}{2}(L_a - L_b)$$

$$L_d = \frac{3}{2}(L_a + L_b)$$

The torque equations may be written as:

$$T_e = \frac{3}{2}(\lambda_d i_q^r - \lambda_q i_d^r)$$

DECIDE THE INITIAL CONDITION

After getting the state space model of the PMSM, the only thing to do is to solve the differential equations in the q-d reference frame but when we proceed to solve the equations, we must know the initial condition by using Rung-Kutta integral method, and the correct initial condition is very important to get the correct result.

Defining $\theta_r = 0$ in (6), we can get the voltage equations in 2-phase stationary reference frame written in matrix form

$$\dot{X} = AX + Bu$$

Where

$$A=\begin{bmatrix} -\frac{R}{Lq} & \frac{Ld}{Lq} \\ \frac{Lq}{Ld}\overline{w}r & -\frac{R}{Ld} \end{bmatrix},$$

$$Bu=\begin{bmatrix} \frac{\overline{w}r\ Ke - v_q^s}{L_q} \\ -\frac{v_d^s}{L_d} \end{bmatrix}$$

There is a symmetrical matrix S in the 2-phase reference frame and we can get the following equation that represents the relationships of the results in different time.

$$i_{qd}^s(\overline{w}_r t+\frac{\pi}{3})=S i_{qd}^s(\overline{w}_r t)$$

Where

$$S=\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$i_{qd}^s=[i_q^s \quad i_d^s]^T$$

The standard non-homogeneous solution of (12) can be written as

$$X(t)=e^{At}X(0)+\int_0^t e^{A(t-\tau)}Bu(\tau)\, d\tau$$

Defining $\lambda(t,t_0)=\int_0^t e^{A(t-\tau)}\, d\tau$, and we have the following expression

$$\lambda(t,t_0)=I(t-\tau_0) + \frac{A}{2!}(t-t_0)^2 + \frac{A}{2!}(t-t_0)^2 \frac{A^2}{3!}(t-t_0)^3 +\cdots+\frac{1}{n!}A^{(n-1)}(t-t_0)^n+\cdots$$

Based on Theory of Matrix [4], we have

$$e^{At}=I+At+\frac{1}{2!}A^2 t^2+\frac{1}{3!}A^3 t^3 +\cdots+\frac{1}{n!}A^n t^n +\cdots$$

From (15) and (16) we can get

$$e^{At}=A\lambda(t,0)+I$$

According to the existing symmetrical relationships of the 3-phase system

$$X(T) = e^{AT}X(0)+\lambda(T,0)Bu=SX(0)$$

From the above equation, the initial condition in the stationary reference frame can be got

$$X(0) = (S - e^{AT})^{-1}\lambda(T,0)Bu=(S-e^{AT})^{-1}A^{-1}(e^{AT}-1)Bu$$

Where

$$T=\frac{\pi}{3\overline{w}r}$$

Being an unsymmetrical structure of the rotor, the above mentioned initial conditions should be transformed to the rotor reference frame in order to proceed the calculation. When the PMSM come to the stead-state, the rotor come to the synchronous speed, and the rotor reference frame have the same speed as the synchronous reference frame, the difference is that there are angular displacement б between two reference frames, showed in Fig. 1. In Fig. 1, q_r - d_r presents the rotor reference frame, q_e-d_e, synchronous reference frame, q_s-d_s, stationary reference frame.

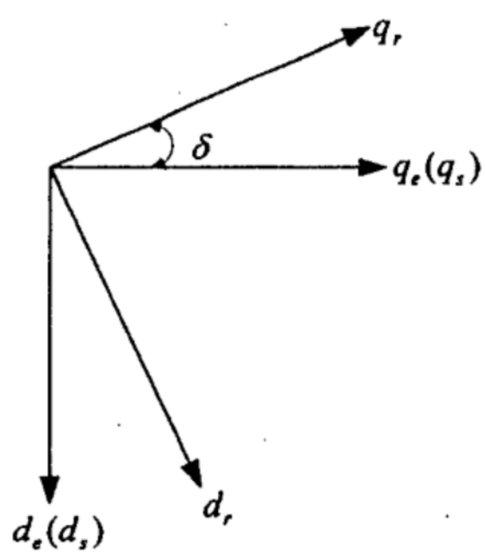

Fig.1 Relationship between two reference frames

Being in the initial state, the synchronous frame coincide with the stationary frame, and the initial conditions in the rotor reference frame can be expressed as:

$$[\begin{matrix} i_q^r \\ i_d^r \end{matrix}]_{t\text{-}0}=[\begin{matrix} \cos(\overline{w}t+б) & -\sin(\overline{w}t+б) \\ \sin(\overline{w}t+\ б) & \cos(\overline{w}t+б) \end{matrix}]_{i=0}[\begin{matrix} i_q^s \\ i_d^s \end{matrix}]$$

Where

$$[i_q^s \quad i_d^s]^T=X(0)$$

The MATLAB language is used to program in the paper , and the Runge-Kutta-Felhberg numerical integral method is used to solve the govern differential equations in the rotor reference frame with one period [0,2pi], and the results of the next several period can be obtained by shifting the calculated result along the time axis . After getting the q-d currents, the torque can be got using (Il), and 3-phase currents can be obtained by using inverse Park's transformation. The flow chart of the simulation is as Fig.2.

The parameters of the PMSM are the same as the paper [l], i.e. P3.4 R, L, = L, = 12.1mH. the electromotive force constant K, =0.083 volt-sec. The input voltage vdc=28 V. Rotor speed equals to 1400 rpm, and 8 = g. The Fig. 3 and Fig.4 are the wave of q-axis current ii and d-axis current i: respectively. The Fig.5 is the wave of a-phase current, and Fig.6, the wave of the output torque

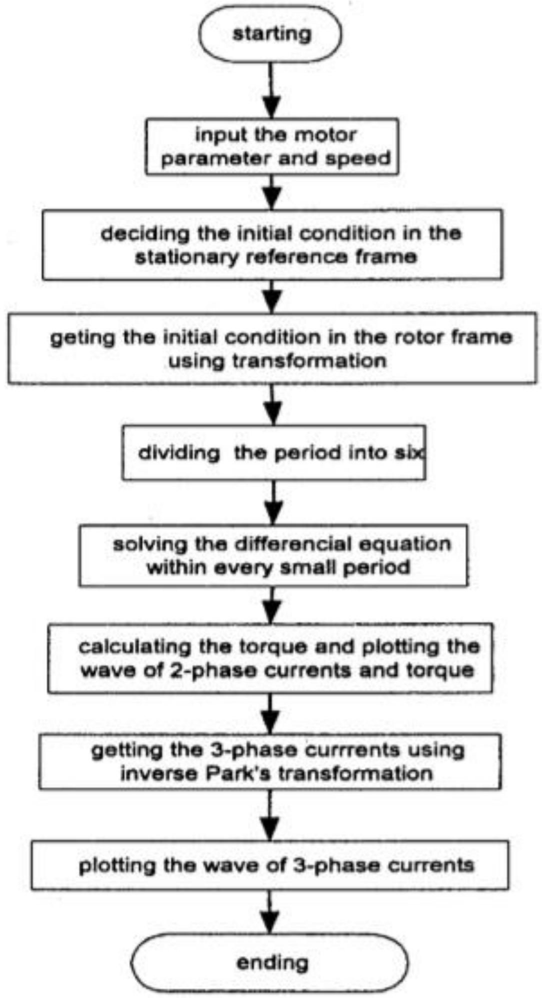

Fig.2 Flow chart of simulation

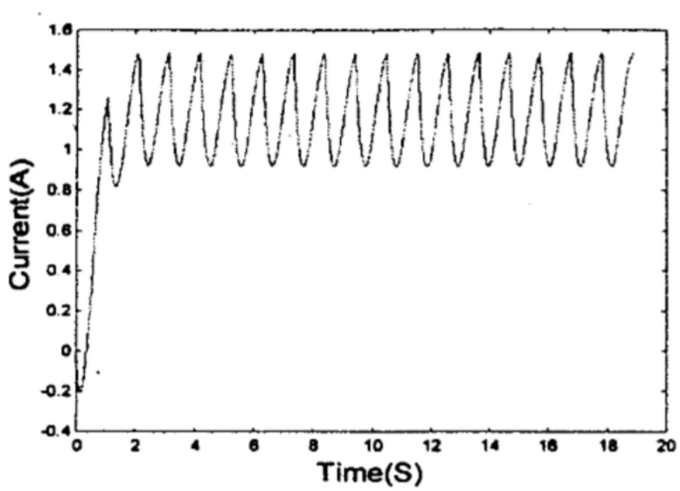

Fig. 3 Wave of q-axis current

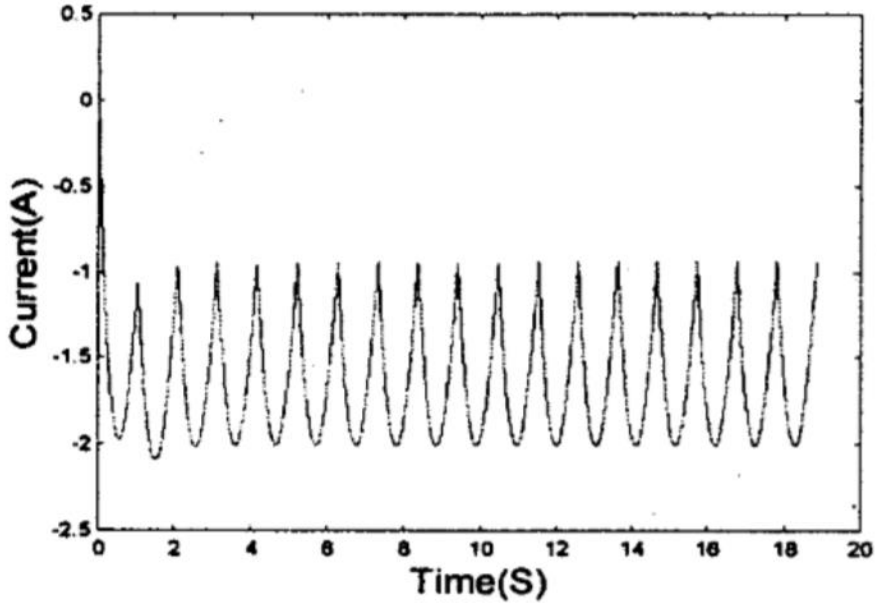

Fig.4 Wave of d-axis current

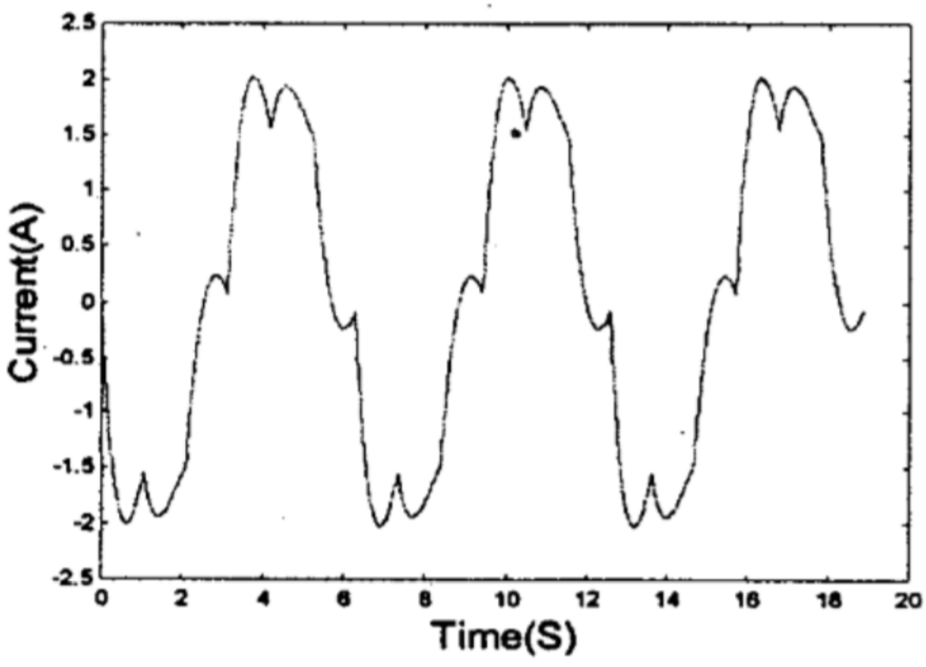

Fig.5 Wave of a-phase current

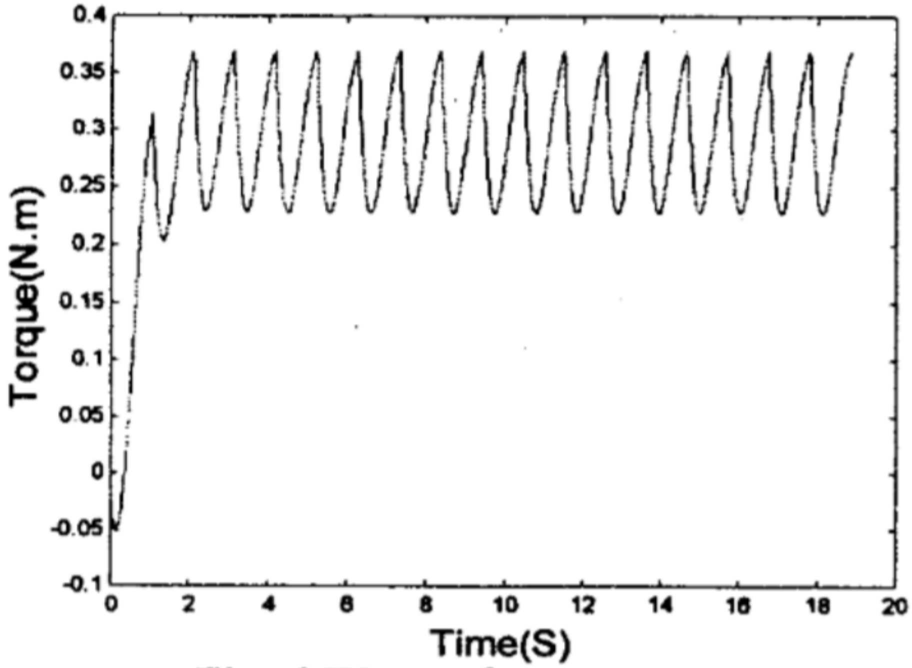

Fig. 6 Wave of output torque

CONCLUSION

This paper addresses the method of modeling and simulation of the PMSM based on state space method. Two transformations of reference frame are used, and the method of deciding the initial conditions is presented. The accurate results can be obtained by using the method in the paper. The MATLAB language is used to program and it avoids choosing integral step by using variable-step Runge-Kutta numerical integral method.