DARE BENEDICT OLUBUKOLA
MECHANICAL ENGINEERING
19/ENG06/016 SERIAL NO. ; 111
MAT 102 ASSIGNMENT

1. Find a unit vector tangent to the space curve $x = t$, $y = t^2$, $z = t^3$ at the point where $t = 1$

Solution

$x = t$, $y = t^2$, $z = t^3$

$r = ti + t^2j + t^3k$

$dr/dt = i + 2tj + 3t^2k$

At $t = 1$

$dr/dt = i + 2j + 3k$

$|dr/dt| = \sqrt{(1)^2 + (2)^2 + (3)^2}$

$= \sqrt{1 + 4 + 9} = \sqrt{14}$

$\therefore T = \dfrac{dr/dt}{|dr/dt|}$

$\Rightarrow T = \dfrac{i + 2j + 3k}{\sqrt{14}}$

2. Given that $A = 4t^3 j + 5k$ and $B = 2t^2 i + 4tj$, if $G = A \times B$. Find the integral of G from $(0 \text{ to } 1)$

Solution

$A = 4t^3 j + 5k$

$B = 2t^2 i + 4tj$

$$G = A \times B = \begin{vmatrix} i & j & k \\ 0 & 4t^3 & 5 \\ 2t^2 & 4tj & 0 \end{vmatrix}$$

$$i \begin{vmatrix} 4t^3 & 5 \\ 4t & 0 \end{vmatrix} - j \begin{vmatrix} 0 & 5 \\ 2t^2 & 0 \end{vmatrix} + k \begin{vmatrix} 0 & 4t^3 \\ 2t^2 & 4t \end{vmatrix}$$

$$i[0 - 20t] - j[0 - 10t^2] + k[0 - 8t^5]$$

$$\therefore G = -20ti + 10t^2 j - 8t^5 k$$

$$\int_0^1 G\,dt = -i\left[\frac{20t^{1+1}}{1+1}\right] + j\left[\frac{10t^{2+1}}{2+1}\right] - k\left[\frac{8t^{5+1}}{5+1}\right]$$

$$= -10t^2 i + \frac{10t^3}{3} j - \frac{8t^6}{6} k$$

$$= \left[-10(1)^2 i + \frac{10(1)^3}{3} j - \frac{8(1)^6}{6} k\right] - \left[-10(0)^2 i + \frac{10(0)^3}{3} j + \frac{8(0)^6}{6} k\right]$$

$$= \left[-10i + \frac{10}{3} j - \frac{8}{6} k\right] - [0]$$

$$\therefore \int_0^1 G\,dt = -10i + \frac{10}{3}j - \frac{8}{6}k$$

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