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MECHANICAL ENGINEERING
19/ENG06/016 SERIAL NO. ; 111
MAT 104 ASSIGNMENT

1. Differentiate $y = \sin\left(\frac{6}{x^2}\right)$ from the first principle.

<u>Solution</u>

$$y = \sin\left(\frac{6}{x^2}\right)$$

$$y + \Delta y = \sin\left[\frac{6}{(x+\Delta x)^2}\right]$$

$$\Delta y = \underbrace{\sin\left(\frac{6}{[x+\Delta x]^2}\right)}_{A} - \underbrace{\sin\left(\frac{6}{x^2}\right)}_{B}$$

$$\Delta y = 2\cos\left(\frac{\frac{6}{(x+\Delta x)^2} + \frac{6}{x^2}}{2}\right)\sin\left(\frac{\frac{6}{(x+\Delta x)^2} - \frac{6}{x^2}}{2}\right)$$

$$\Delta y = 2\cos\left(\frac{\frac{6x^2 + 6(x+\Delta x)^2}{x^2(x+\Delta x)^2}}{2}\right)\sin\left(\frac{\frac{6x^2 - 6(x+\Delta x)^2}{x^2(x+\Delta x)^2}}{2}\right)$$

$$\Delta y = 2\cos\left(\frac{6x^2 + 6x^2 + 12x\Delta x + 6\Delta x^2}{2x^2(x+\Delta x)^2}\right)\sin\left(\frac{6x^2 - 6x^2 - 12x\Delta x - 6\Delta x^2}{2x^2(x+\Delta x)^2}\right)$$

$$\Delta y = 2\cos\left(\frac{12x^2 + 12x\Delta x + 6\Delta x^2}{2x^2(x+\Delta x)^2}\right)\sin\left(\frac{-12x\Delta x - 6\Delta x^2}{2x^2(x+\Delta x)^2}\right)$$

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\left(\frac{\cancel{6}^{3}[2x^2+2x\Delta x+\Delta x^2]}{\cancel{2}x^2(x+\Delta x)^2}\right)\sin\left(\frac{-\cancel{6}^{3}\Delta x[2x+\Delta x]}{\cancel{2}x^2(x+\Delta x)^2}\right)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = 2\cos\left(\frac{3[2x^2+2x\Delta x+\Delta x^2]}{x^2(x+\Delta x)^2}\right)\cdot\frac{\sin\left(\frac{-3\Delta x[2x+\Delta x]}{x^2(x+\Delta x)^2}\right)}{\Delta x}$$

Multiply through by $\frac{-3[2x+\Delta x]}{x^2(x+\Delta x)^2}$

$$\Rightarrow \frac{\Delta y}{\Delta x} = 2\cos\left(\frac{3(2x^2+2x\Delta x+\Delta x^2)}{x^2(x+\Delta x)^2}\right)\frac{\sin\left(\frac{-3\Delta x[2x+\Delta x]}{x^2(x+\Delta x)^2}\right)}{\frac{-3\Delta x[2x+\Delta x]}{x^2(x+\Delta x)^2}}\cdot\frac{-3[2x+\Delta x]}{x^2(x+\Delta x)^2}$$

Take $\lim\limits_{\Delta x\to 0}$ 4 both sides;

Note; $\lim\limits_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x}\longrightarrow 1$

$$\therefore\ \frac{dy}{dx} = 2\cos\left(\frac{3(2x^2+0)}{x^2(x+0)^2}\right)\cdot 1\cdot\frac{-3(2x+0)}{x^2(x+0)^2}$$

$$\frac{dy}{dx} = 2\cos\frac{6x^2}{x^4}\cdot\frac{-6x}{x^4}$$

$$\therefore \frac{dy}{dx} = \frac{-6}{x^3} \cdot 2\cos\frac{6}{x^2}$$

2. Find the area under the curve, given parametric equations $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$, at $t = 1$ and $t = 3$.

Solution

$$A = \int_a^b y \, dx$$

$$x = 4t^3 - t^2 \quad , \quad y = t^4 + 2t^2$$

$$dx/dt = 12t^2 - 2t$$

$$dx = (12t^2 - 2t)\,dt$$

$$\therefore A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)\,dt$$

$$= \int_1^3 12t^6 - 2t^5 + 24t^4 - 4t^3 \, dt$$

$$= \left[\frac{12t^{6+1}}{6+1} - \frac{2t^{5+1}}{5+1} + \frac{24t^{4+1}}{4+1} - \frac{4t^{3+1}}{3+1} \right]_1^3$$

$$= \left[\frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4} \right]_1^3$$

$$= \left[\frac{12(3)^7}{7} - \frac{(3)^6}{3} + \frac{24(3)^5}{5} - \frac{\cancel{4}(3)^4}{\cancel{4}} \right] - \left[\frac{12(1)^7}{7} - \frac{(1)^6}{3} + \frac{24(1)^5}{5} - \frac{\cancel{4}(1)^4}{\cancel{4}} \right]$$

$$A = \left[\frac{26244}{7} - \frac{729}{3} + \frac{5832}{5} - 81\right] - \left[\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1\right]$$

$$A = \frac{160704}{35} - \frac{544}{105}$$

$A = 4586.36$ square units

3. If $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$, Find dy/dx.

Solution

$dx/dt = 12t^2 - 2t$; $\frac{dy}{dt} = 4t^3 + 4t$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{4t^3 + 4t}{12t^2 - 2t}$$

$$\frac{dy}{dx} = \frac{\cancel{4t}(t^2+1)}{\cancel{2t}(6t-1)} = \frac{2(t^2+1)}{(6t-1)}$$