Modelling a Permanent Magnet Synchronous Motor (PMSM)

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Abstract

In recent years, Permanent Magnet Synchronous Motors (PMSM) have gained a lot of attention due to their high efficiency and accuracy in high precision applications. This paper briefly discusses the PMSM, with its differences to a conventional synchronous motor. It then gives a deep dive into two methods of modelling a PMSM, which are, the Parks transformation and the embedded phase domain model.

1. Introduction

Having a good knowledge of electrical machines is a prerogative for the successful work of electrical engineers. Knowledge of the equivalent schemes in steady states and mechanical characteristics is required for selecting a machine which would be adequate for a particular application, for designing systems containing electrical machines, as well as for solving the problems which may arise in industry and power engineering. Knowledge of the dynamic model of electrical machines is necessary for solving the control problems of generators and motors, for designing protection and monitoring systems, for determining the structures and control parameters in robotics, as well as for solving the problems in automation of production, electrical vehicles, and other similar applications (Slovodan, 2013).

Motors are electrical machines that convert electrical energy into mechanical energy. The major classifications of AC motors are asynchronous and synchronous motors. Asynchronous motors are singly excited machines, that is, stator windings are connected to an AC supply which create a rotating magnetic field which turns the rotor via mutual induction. They are also known as induction machines. On the other hand, a synchronous motor requires AC supply for the stator windings and DC supply for the rotor windings (Balashanmugham *et al*, 2016). The motor speed is determined by the AC supply and the number of poles of the synchronous motor. Synchronous motors are capable of running at constant speed irrespective of the load acting on them. They are highly efficient and used in high precision application. Synchronous machines can be classified depending on the rotor that produces its magnetic field. One of these classifications is the Permanent Magnet Synchronous Motors (PMSM).

In recent years, Permanent Magnet Synchronous Motors (PMSMs) are increasingly applied in several areas such as traction, automobiles, robotics and aerospace technology (Bowen *et al*, 2012). The PMSM consists of conventional three phase windings in the stator and permanent magnets in the rotor. The purpose of the field windings in the conventional synchronous machine is done by permanent magnets in PMSM. The conventional synchronous machine requires AC and DC supply, whereas the PMSM requires only AC supply for its operation. One of the greatest advantages of PMSM over its counterpart is the removal of dc supply for field excitation.

This paper presents two different approaches to model the permanent magnet synchronous machine in RTDS, the traditional dq0 model (which is also known as Parks Transform) of the machine and the embedded phase domain model.

2. Parks Transform Modeling of PMSM

To model a PMSM using Parks Transform method there are two steps involved which are acquiring the operating equations of a PMSM and applying the Parks Transformation to the equations

2.1 Acquiring Operating Equations

The motor axis has been developed using d-q rotor reference frame theory as shown in Figure 2.1. At any particular time, t, the rotor reference axis makes an angle θ_r with the fixed stator axis and the rotating stator mmf creates an angle α with the rotor d axis (Balashanmugham *et al*, 2016). It is viewed that at any time t, the stator mmf rotates at the same speed as that of the rotor axis. The following assumptions are made for the modeling of the PMSM without damper windings.

i. Saturation is neglected
ii. The induced EMF is sinusoidal
iii. Hysteresis and eddy current losses are negligible
iv. There is no field current dynamics

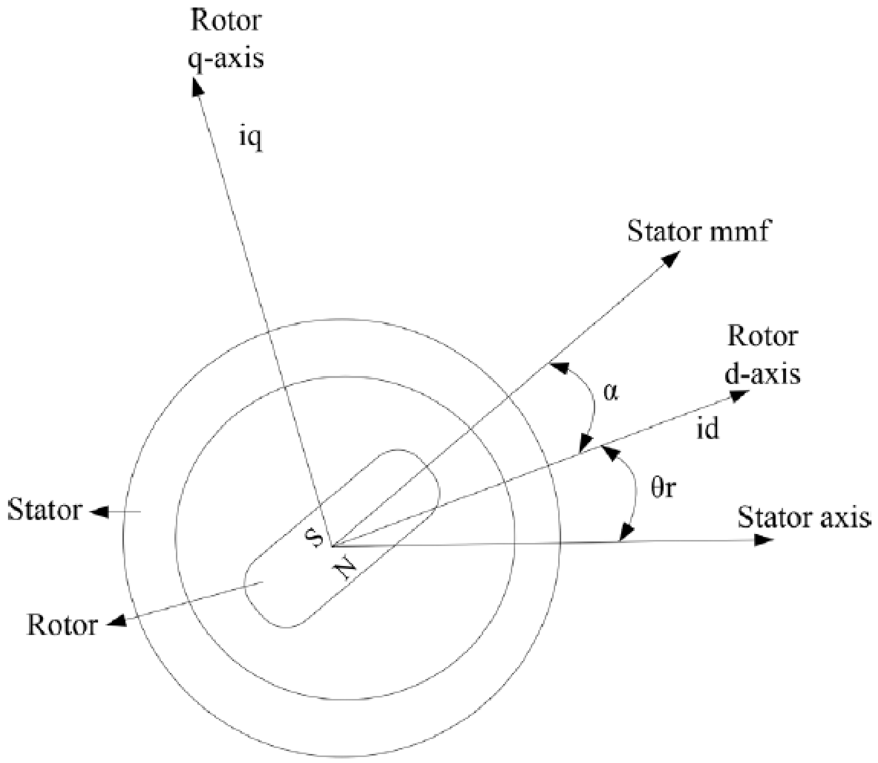

Figure 2.1. Motor Axis (Balashanmugham *et al*, 2016)

Voltage equations from the model are given by,

$$V_q = R_s i_q + \omega_r \lambda_d + \rho \lambda_q \qquad (1)$$

$$V_d = R_s i_d - \omega_r \lambda_q + \rho \lambda_d \qquad (2)$$

Flux linkages are given by,

$$\lambda_q = L_q i_q \qquad (3)$$

$$\lambda_q = L_q i_q + \lambda_f \qquad (4)$$

Substituting Eq. (3) and Eq. (4) into Eq. (1) and Eq. (2)

$$V_q = R_s i_q + \omega_r (L_d i_d + \lambda_f) + \rho L_d i_d \qquad (5)$$

$$V_d = R_s i_d - \omega_r L_q i_q + \rho (L_d i_d + \lambda_f) \qquad (6)$$

Arranging Eq. (5) and Eq. (6) in matrix form,

$$\begin{bmatrix} V_q \\ V_d \end{bmatrix} = \begin{bmatrix} R_s + \rho L_q & \omega_r L_d \\ -\omega_r L_q & R_s + \rho L_d \end{bmatrix} \begin{bmatrix} i_q \\ i_d \end{bmatrix} + \begin{bmatrix} \omega_r \lambda_f \\ \rho \lambda_f \end{bmatrix} \qquad (7)$$

The developed torque motor is being given by,

$$T_e = \frac{3}{2}\left(\frac{P}{2}\right)(\lambda_d i_q - \lambda_q i_d) \qquad (8)$$

The mechanical torque equation is,

$$T_e = T_L + B\omega_m + J\frac{d\omega_m}{dt} \qquad (9)$$

Solving for the rotor mechanical speed form Eq. (9)

$$\omega_m = \int \left(\frac{T_e - T_L - B\omega_m}{J}\right) dt \qquad (10)$$

and,

$$\omega_m = \omega_r \left(\frac{2}{P}\right) \qquad (11)$$

In the above equations ω_r is the rotor electrical speed, ω_m is the rotor mechanical speed.

2.2 Applying Parks Transformation

The dq0 equivalent circuit of the PM machine shown in Fig 2.2. is similar to the one for the synchronous machine

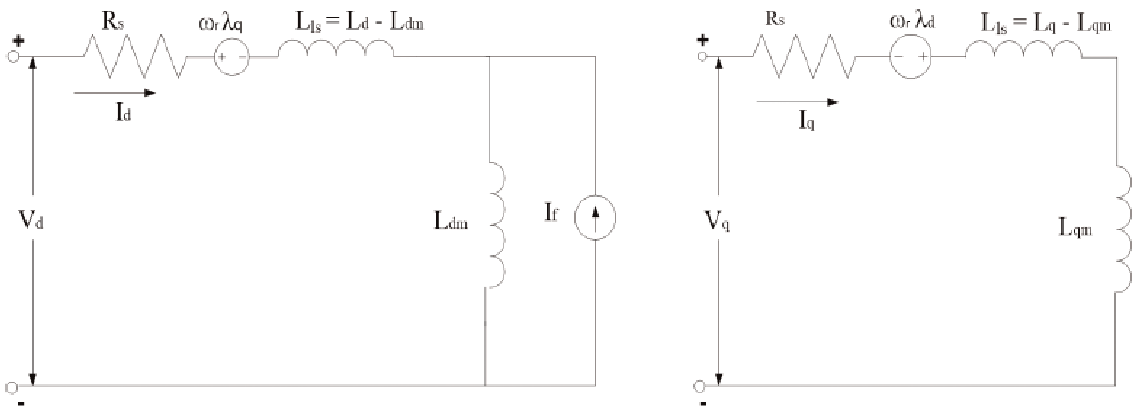

Figure 2.2. Equivalent circuit of PMSM without damper windings (Balashanmugham *et al*, 2016)**.**

The dynamic d-q modelling of the system is used for the study of motor during transient state and as well as in the steady state conditions. It is achieved by converting the three phase voltages and currents to dqo axis variables by using the Parks transformation. Converting the phase voltages variables Vabc to Vdqo variables in rotor reference frame axis are illustrated in the equations,

$$\begin{bmatrix} V_q \\ V_d \\ V_o \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos(\theta_r - 120) & \cos(\theta_r + 120) \\ \sin\theta_r & \sin(\theta_r - 120) & \sin(\theta_r + 120) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad (12)$$

Convert V_{dqo} to V_{abc}

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \sin\theta_r & 1 \\ \cos(\theta_r - 120) & \sin(\theta_r - 120) & 1 \\ \cos(\theta_r + 120) & \sin(\theta_r + 120) & 1 \end{bmatrix}\begin{bmatrix} V_q \\ V_d \\ V_o \end{bmatrix} \qquad (13)$$

3. Embedded Phase Domain Model

The machine can be modeled as set of mutual inductances that change in value with time. In this case the model doesn't have the problem of interface that may cause numerical instabilities. This model does not use the Park transform and directly solves the machine equations in phase domain (Dehkordi *et al*., 2005). For a machine, or in general, a set of time-varying mutual inductances can be written:

$$v(t) = \frac{d}{dt}\left([l(t)] \cdot i(t)\right) \quad (14)$$

Where v and i are vectors of node voltages and branch currents and [L] is the inductance matrix of the set. Using the trapezoidal integration, we have:

$$i(t) = \frac{\Delta t}{2}[L(t)]^{-1} \cdot v(t) + \frac{\Delta t}{2}[L(t)]^{-1} + v(t-\Delta t) + [L(t)]^{-1}[L(t-\Delta t)]i(t-\Delta t) \quad (15)$$

where,

$$I_h = \frac{\Delta t}{2}[L(t)]^{-1} + v(t-\Delta t) + [L(t)]^{-1}[L(t-\Delta t)]i(t-\Delta t)$$

The machine can be modelled as set of current sources I_h parallel to a network of g values that can be obtained from the matrix $GL = \frac{\Delta t}{2}[L]^{-1}$

3.1. Calculating the Phase Domain Inductances of the Permanent Magnet Machine

Equation (15) can be directly used to model the machine; however, we need to have the value of the inductances as a function of time. Using an orthogonal transformation; $T^{-1}(\theta) = T^{t}(\theta)$ a matrix is given:

$$\left[L_{a_b c}(\theta)\right] = T^{-1}(\theta) = \begin{bmatrix} L_d & 0 & 0 \\ 0 & Lq & 0 \\ 0 & 0 & L_0 \end{bmatrix} \cdot T(\theta) \quad (16)$$

$$\begin{bmatrix} L_{af}(\theta) \\ L_{bf}(\theta) \\ L_{cf}(\theta) \end{bmatrix} = T^{-1}(\theta) \begin{bmatrix} L_{md} \\ 0 \\ 0 \end{bmatrix} \qquad (17)$$

From (16) and (17) as an example we have:

$$L_{aq}(\theta) = l_s + l_m \cos 2\theta (H) \qquad (18)$$

$$L_{ab}(\theta) = L_{ba}(\theta) = -M_s - l_m \cos 2\left(\theta + \frac{\pi}{6}\right)(H) \qquad (19)$$

$$L_{af}(\theta) = L_{fa}(\theta) = M_f \cos \theta (H) \qquad (20)$$

The rest of inductances can be calculated in a similar way. The self-inductance of the field is the same in phase domain and dq0 domain:

$$L_f(\theta) = L_f \qquad (21)$$

Where:

$$L_s = \frac{1}{3}(L_o + L_d + L_q)$$

$$L_m = \frac{1}{3}(L_d - L_q)$$

$$M_s = -\frac{1}{3}(\frac{-L_d + L_q}{2} + L_o)$$

$$M_f = \sqrt{\frac{2}{3}} L_{md}$$

3.2. Inverting the Inductance Matrix of the Machine

The inductance matrix[L] of the machine has to be inverted in every time step to calculate the G matrix of the machine

4. Conclusion

This paper gave an overview of the need for modelling electrical machines, a brief description of Permanent Magnet Synchronous Motors (PMSM) and has addressed the methods of modelling the PMSM based on Parks transforms and embedded phase domain model.

5. References

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