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① $\int e^x \sin x \, dx$

$u = \sin x$, $dv = e^x$
$du = \cos x \, dx$; $v = e^x$

$\int u \, dv = uv - \int v \, du$

$\sin x (e^x) - \int e^x \cos x \, dx$

$e^x \sin x - \int e^x \cos x \, dx$

$u = \cos x \quad dv = e^x$
$du = -\sin x \, dx \quad v = e^x$
$\cos x (e^x) - \int e^x (-\sin x) \, dx$
$e^x \cos x + \int e^x \sin x \, dx$

$e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

Let $I = \int e^x \sin x \, dx$

$I = e^x \sin x - e^x \cos x - I$

$2I = e^x \sin x - e^x \cos x$

$$I = \frac{e^x \sin x - e^x \cos x}{2}$$

$$\therefore \int e^x \sin x \, dx = \frac{e^x \sin x - e^x \cos x}{2} + C$$

$$\int e^x \sin x \, dx = \frac{1}{2}\left[e^x \sin x - e^x \cos x\right] + C$$

② $\int 2x^2 \ln x \, dx$

$u = \ln x \qquad dv = 2x^2$

$\frac{du}{dx} = \frac{1}{x} \qquad v = \frac{2}{3}x^3$

$du = x^{-1} dx$

$\int u\,dv = uv - \int v\,du$

$= \frac{2}{3}x^3 \ln x - \int \frac{2}{3}x^3 \cdot x^{-1} dx$

$= \frac{2}{3}x^3 \ln x - \frac{2}{3}\int x^3 \cdot x^{-1} dx$

$= \frac{2}{3}x^3 \ln x - \frac{2}{3}\int x^2 dx$

$= \frac{2}{3}x^3 \ln x - \frac{2}{3}\left[\frac{x^{2+1}}{2+1}\right]$

$= \frac{2}{3}x^3 \ln x - \frac{2}{3}\left[\frac{x^3}{3}\right]$

$= \frac{2}{3}x^3 \ln x - \frac{2x^3}{9}$

$\therefore \int 2x^2 \ln x \, dx = \frac{2}{3}x^3\left[\ln x - \frac{1}{3}\right] + c$

③ $\int x^2 \sin x \, dx$

$u = x^2 \qquad dv = \sin x$

$du = 2x\,dx \qquad v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$= x^2(-\cos x) - \int(-\cos x)(2x\,dx)$

$= -x^2 \cos x - \int -2x \cos x \, dx$

$= -x^2 \cos x + 2\int x \cos x$

$u = x \qquad dv = \cos x$

$\frac{du}{dx} = 1 \quad \therefore du = dx \qquad v = \sin x$

$x \sin x - \int \sin x \, dx$

$x \sin x - (-\cos x)$

$x \sin x + \cos x$

$\int x^2 \sin x \, dx = -x^2 \cos x + 2[x \sin x + \cos x] + C$

$\therefore \int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2\cos x + C$

(4) $\int x \cos x \, dx$

$u = x$, $dv = \cos x$

$\frac{du}{dx} = 1$ $\quad v = \sin x$

$\therefore du = dx$

$$\int u\,dv = uv - \int v\,du$$

$$= x(\sin x) - \int \sin x \, dx$$

$$= x \sin x + \cos x + C$$