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Assignment

i) $\int e^x \sin x \, dx$

Solution

$u = \sin x \qquad dv = e^x$

$du = \cos x \, dx \qquad v = e^x$

$= (\sin x)(e^x) - \int e^x \cos x \, dx$

$= e^x \sin x - \int e^x \cos x \, dx$

$\int e^x \cos x \, dx$

$u = \cos x \qquad dv = e^x$

$du = -\sin x \, dx \qquad v = e^x$

$\int u \, dv = uv - \int v \, du$

$= e^x \cos x - \int e^x (-\sin x \, dx$

$= e^x \cos x + \int e^x (\sin x \, dx)$

$\int e^x \sin x \, dx = e^x \sin x - [e^x \cos x + \int e^x \sin x \, dx]$

$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

Let $\int e^x \sin x \, dx = I$

$I = e^x \sin x - e^x \cos x - I$

$I + I = e^x \sin x - e^x \cos x$

$\frac{2I}{2} = \frac{e^x \sin x}{2} - \frac{e^x \cos x}{2}$

$I = \frac{e^x \sin x}{2} - \frac{e^x \cos x}{2}$

$\int e^x \sin x \, dx = \frac{e^x \sin x - e^x \cos x}{2} + c$

$\int e^x \sin x \, dx = \frac{1}{2}[e^x \sin x - e^x \cos x] + c$

2 $\int 2x^2 \ln x \, dx$

Solution

$2\int x^2 \ln x$

$u = \ln x \qquad dv = x^3$

$du = 1/x \, dx \qquad v = x^3/3$

$\int u\,dv = uv - \int v\,du$

$= 2\left[\ln(x) \times \frac{x^3}{3} - \int \frac{x^3}{3} \times \frac{1}{x} \, dx\right]$

$2\left[\frac{x^3 \ln(x)}{3} - \int \frac{x^2}{3} \, dx\right]$

$2\left[\frac{x^3 \ln(x)}{3} - \frac{1}{3}\int x^2 \, dx - \right]$

$2\left[\frac{x^2 \ln(x)}{3} - \frac{1}{3} \times \frac{x^3}{3}\right]$

$2\left[\frac{x^3 \ln(x)}{3} - \frac{x^3}{9}\right]$

$\therefore \int 2x^2 \ln x \, dx = \left[\frac{2}{3} x^3 \left(\frac{\ln(x)}{1} - \frac{1}{3}\right)\right] + C$

3) $\int x^2 \sin x \, dx$

$\therefore u = x^2 \quad dv = \sin x \mid du = 2x\,dx \cdot \quad V = -\cos x$

$\int u\,dv = uv - \int v\,du$

$= x^2(-\cos x) - \int(-\cos x) \times 2x\,dx$

$= -x^2 \cos x + \int 2x \cos x \, dx$

$\int 2x \cos x \, dx$

$u = 2x \qquad dv = \cos x$

$du = 2dx \qquad v = \sin x$

$\int u\,dv = uv - \int v\,du$

$= 2x \sin x - \int \sin x \times 2\,dx$

$= 2x \sin x - 2\int \sin x \, dx$

$= 2x \sin x - 2(-\cos x)$

$= 2x \sin x + 2\cos x$

$\therefore \int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2\cos x + C$

1) $\int x \cos x \, dx$

Solution

$u = x$ $\quad$ $dv = \cos x$

$du = 1 dx$ $\quad$ $v = \sin x$

$\int u\,dv = uv - \int v\,du$

$= x\sin x - \int \sin x \cdot 1 dx$

$= x\sin x - \int \sin x\, dx$

$= x \sin x - (-\cos x) + C$

$= x \sin x + \cos x + C$