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Modelling the Operation of a Permanent-Magnet Synchronous Machine

Abstract: Permanent-Magnet Synchronous Machines have large applications in various fields having better efficiency and lower cost than induction motors. It consists of a stator and rotating part (rotor) requiring only AC supply for its operation. Their operation is similar to that of normal synchronous motors with the stator creating the magnetomotive force. The rotor flux has the course and direction determined by the rotor's position. Both the stator flux vector and rotor flux vector rotate at the rotor speed. In modelling this machine, some assumptions are made, one of which is that the damper windings are not considered. The dynamic model is developed using the direct (d) axis and quadrature (q) axis. Park's transformation was used to develop the matrix equations

Keywords: Park's transformation, damper windings, direct-axis, quadrature-axis

Introduction

This type of machines (Synchronous machines) have its rotor speed and speed of the rotating stator generated magnetic field synchronised. They have applications in systems requiring speed reversals and wide-range power variations. The stator is composed of three identical winding distributed in space such that any two successive windings have a space of 120° between them. A turning field is generated along the air-gap between the stator and the rotor when the stator windings are current fed by a balanced 3-phase AC supply. The rotor however needs to be excited separately to rotate. (Note, 2020) Synchronous machines constitute of both synchronous motors and generators.

Synchronous machines' operation depends on the following equation below.

$$Ns = \frac{120f}{P}$$

Or

$$f = \frac{PNs}{120}$$

Ns = Synchronous speed in revolution per minute (r.p.m)

f = supply frequency

p = number of poles of the machine

The above equation is always maintained when it is connected to an electric power system. (Globe, n.d.)

Permanent magnet synchronous machines have better efficiency and power/mass ration than induction motors, with a much lower material cost.(Hadziselimovic & Stumberger, 2005)

The inductors in these machines consists of permanent magnets. A permanent magnet synchronous motor consists of a fixed stator and rotating part (rotor). The permanent magnets in the rotor have high coercive force. The rotors are of either salient pole or non-salient pole. The former has equal direct and quadrature inductances while the latter has unequal direct and quadrature inductances.

The stator has an outer frame and a core with windings. Based on stator design, Permanent magnet synchronous machines are divided into stators with distributed winding and stators with concentrated windings. The distributed winding have number of slots per pole and phase Q = 2, 3…, k. While the concentrated have number of slots per pole and phase Q = 1. (Engineeringsolutions, 2020)

The permanent magnet synchronous machines (PMSM) requires only AC supply for its operation. PMSM can be classified based on the direction of field flux into Radial field and axial field. The flux direction is along the radius of the machine in radial field. They are most commonly used. In axial field however, the flux direction is parallel to the rotor shaft. (Balashanmugham & Maheswaran, n.d.) (Engineeringsolutions, 2020)

Synchronous machines being an electrical transducer converting mechanical energy into electrical energy and vice versa obey the Law of Electromagnetic induction and the Law of interaction. They require external means to bring their speed close to synchronous speed for synchronisation to occur.

Operation of a synchronous machine

Permanent magnet synchronous machines have the same operating and performance characteristics as synchronous machines. PMSCs have AC currents in stator windings. The stator currents create the stator's magnetomotive force Fs revolving at the synchronous speed $\frac{\omega e}{P}$. The synchronous speed depends upon the angular frequency ωe of the stator currents and the number of pole pairs p. The stator's magnetomotive force creates the stator flux $\phi s = Fs/R^{\varphi}$ which depends in the magnetic resistance R^{φ}. The flux rotates at the same speed as the magnetomotive force. Rotor in synchronous machines may have permanent magnets built into magnetic circuit or excitation windings supplied by DC current. The rotor flux has the course and direction determined by the rotor's position. Both the stator flux vector and rotor flux vector rotate at the rotor speed. Their relative position and angle of separation is constant. The torque and power is dependent on these two vectors. (*Power Electronics and Power Systems*, n.d.)

Modelling the PMSC

Nomenclature

fc crossover frequency
id d-axis current
Ldm d-axis magnetizing inductance
L_d d-axis self-inductance
V_d d-axis voltage
ρ derivative operator
T_e develop electromagnetic torque
d direct or polar axis
ω_r electrical speed
i_f equivalent permanent magnet field current
L_s equivalent self-inductance per phase
λ_d flux linkage due d axis
λ_q flux linkage due q axis
λ_f PM flux linkage or field flux linkage
k_p proportional control gain
i_q q-axis current
L_{qm} q-axis magnetizing inductance
L_q q-axis self-inductance
V_q q-axis voltage
q quadrature or interpolar axis
λ_{dm} flux linkage due to rotor magnets linking the stator B friction
J inertia
k_i integral control gain
T_L load torque
ω_{rated} motor rated speed
T_m motor torque
P number of poles
I_m peak value of supply current
ω_m rotor speed
θ_r rotor position
L self-inductance
L_{ls} stator leakage inductance
R_s stator resistance
ia, ib, ic three phase currents
V_a, V_b, V_c three phase voltage

T_{ref} reference motor torque

The mathematical model of a PMSC is based on the following assumptions

1. The stator windings are positioned sinusoidally along the air-gap as far as the mutual effect with the rotor is concerned.
2. The stator slots cause no significant variations of the rotor inductance with rotor position.
3. Magnetic hysteresis and saturation effects are negligible.
4. The stator windings are considered symmetrical.
5. Damper windings are not considered.
6. Capacitance of all the windings is neglected.
7. Resistances of the coils are assumed to be constant. (Apte, Walambe, Joshi, Rathod, & Kolhe, 2014), (Balashanmugham & Maheswaran, n.d.)

The dynamic model of a PMSC is derived from a two-phase synchronous reference, direct (d) and quadrature (q) axis. The q-axis is 90^o ahead of the d-axis with respect to the direction of rotation.

In modelling of a PMSC, the transient and steady-state behaviour in electrical and mechanical subsystems must be considered. The Electrical properties involve the steady state and transient relations between the machine voltages, currents and flux linkages. Mechanical properties have to do with the rotor speed, electromagnetic torque and motion resistance. (*Power Electronics and Power Systems*, n.d.)

Using the Park's transformation: Using a three phase synchronous machine with voltages Va, Vb, Vc in the direct (d-axis) axis and quadrature (q-axis) axis, the transformation is given as:

$$Vdq0 = T * Vabc \text{ ---- (1)}$$

Where

$$[T] = \frac{2}{3}\begin{bmatrix} cos\theta r & \cos(\theta r - \frac{2\pi}{3}) & \cos(\theta r + \frac{2\pi}{3}) \\ -sin\theta r & -sin(\theta r - \frac{2\pi}{3}) & -sin(\theta r + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \text{ ---- (2)}$$

OR

$$[T] = \frac{2}{3}\begin{bmatrix} cos\theta r & \cos(\theta r - \frac{2\pi}{3}) & \cos(\theta r + \frac{2\pi}{3}) \\ sin\theta r & sin(\theta r - \frac{2\pi}{3}) & sin(\theta r + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \text{---- (3)}$$

Calculating for phase voltages

$$\text{Vabc} = \text{T}^{-1} * \text{Vdq0} \text{ ---- (4)}$$

Calculating T^{-1}

$$T^{-1} = \begin{bmatrix} cos\theta r & sin\theta r & 1 \\ cos(\theta r - \frac{2\pi}{3}) & sin(\theta r - \frac{2\pi}{3}) & 1 \\ cos(\theta r + \frac{2\pi}{3}) & sin(\theta r + \frac{2\pi}{3}) & 1 \end{bmatrix} \text{---- (5)}$$

Using stator equations, the voltages at d-axis and q-axis is expressed as:

$$V_d = \text{Rsid} - \omega\text{rLqiq} + \rho\ (\text{Ldid} + \lambda\text{f}\,) \text{ ---- (6)}$$

$$V_q = \text{Rsiq} + \omega\text{r}\ (\text{Ldid} + \lambda\text{f}) + \rho L_d i_d \text{ ---- (7)}$$

In Matrix form

$$\begin{bmatrix} Vq \\ Vd \end{bmatrix} = \begin{bmatrix} \text{Rs} + \rho\text{Lq} & \omega\text{rLd} \\ -\omega\text{rLq} & \text{Rs} + \rho\text{Ld} \end{bmatrix} \begin{bmatrix} \text{iq} \\ \text{id} \end{bmatrix} + \begin{bmatrix} \omega\text{r}\lambda\text{f} \\ \rho\lambda\text{f} \end{bmatrix} \text{---- (8)}$$

The developed torque equation for a PMSC is derived as

$$Te = \frac{3}{2}\left(\frac{P}{2}\right)(\lambda\text{diq} - \lambda\text{qid}) \text{ ---- (9)}$$

$$Te = Tl + \text{B}\omega\text{m} + \text{J}\frac{\text{d}\omega\text{m}}{dt} \text{ ---- (10)}$$

Solving for ωm

$$\omega\text{m} = \int \left(\frac{\text{Te} - \text{TL} - \text{B}\omega\text{m}}{\text{J}}\right) dt \text{ ---- (11)}$$

$$\omega\text{m} = \omega\text{r}\left(\frac{2}{P}\right) \text{ ---- (12)}$$

Substituting equation (8) into equation (4)

$$\begin{bmatrix} Va \\ Vb \\ Vc \end{bmatrix} = \frac{2}{3}\begin{bmatrix} cos\theta r & sin\theta r & 1 \\ cos(\theta r - \frac{2\pi}{3}) & sin(\theta r - \frac{2\pi}{3}) & 1 \\ cos(\theta r + \frac{2\pi}{3}) & sin(\theta r + \frac{2\pi}{3}) & 1 \end{bmatrix}\left[\begin{bmatrix} Rs + \rho Lq & \omega rLd \\ -\omega rLq & Rs + \rho Ld \end{bmatrix}\begin{bmatrix} iq \\ id \end{bmatrix} + \begin{bmatrix} \omega r\lambda f \\ \rho\lambda f \end{bmatrix}\right] \text{ ---- (13)}$$

OR

$$\begin{bmatrix} Va \\ Vb \\ Vc \end{bmatrix} = \frac{2}{3}\begin{bmatrix} cos\theta r & sin\theta r & 1 \\ cos(\theta r - \frac{2\pi}{3}) & sin(\theta r - \frac{2\pi}{3}) & 1 \\ cos(\theta r + \frac{2\pi}{3}) & sin(\theta r + \frac{2\pi}{3}) & 1 \end{bmatrix}\begin{bmatrix} Vq \\ Vd \\ Vo \end{bmatrix}$$

Solving for Vdqo

$$\begin{bmatrix} Vq \\ Vd \\ Vo \end{bmatrix} = \frac{2}{3}\begin{bmatrix} cos\theta r & cos(\theta r - \frac{2\pi}{3}) & cos(\theta r + \frac{2\pi}{3}) \\ sin\theta r & sin(\theta r - \frac{2\pi}{3}) & sin(\theta r + \frac{2\pi}{3}) \\ 0.5 & 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} Va \\ Vb \\ Vc \end{bmatrix} \text{ ---- (14)}$$

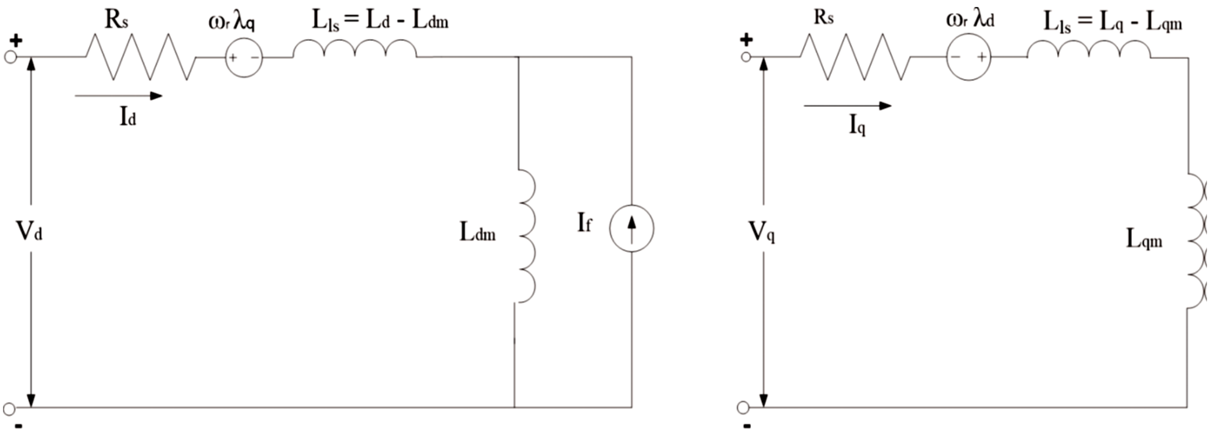

Figure 1: Permanent magnet synchronous machine circuit (motor)

The d and q-axis currents in the frequency (s) domain can be expressed as

$$\text{Id} = \frac{(-V_d - R_s I_d - \omega_r)}{sL_q} \text{ ---- (15)}$$

$$\text{Iq} = \frac{(-V_q - R_s I_q - \omega_r (L_d + L_{ls}) I_d + \omega_r \rho_r}{sL_q} \text{ ---- (16)}$$

Result and Conclusion

The electromagnetic output torque, mechanical speed and stator current are collectively dependent on the number of pole pairs, d-axis inductance, q-axis inductance, inertia, friction coefficient and flux (considering a motor). While the voltage, current and power outputs are dependent on the mechanical input torque (considering a generator). Using Park's transformation, the electrical and mechanical equations of a permanent magnet synchronous machine is determined. These equations are their relations can be used for simulation using any digital signal processing software/simulation software.

References

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