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Mechatronics Engineering

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Math 104 assignment

(1) $\int e^x \sin x dx$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Let $u = e^x$ and $\frac{dv}{dx} = \sin x$

$$\frac{du}{dx} = e^x$$

$$v = -\cos x$$

$$\begin{aligned} \int e^x \sin x dx &= e^x (-\cos x) - \int (-\cos x) e^x dx \\ &= -e^x \cos x + \int e^x \cos x dx \end{aligned}$$

$$\int e^x \cos x dx$$

let $u = e^x$ and $\frac{dv}{dx} = \cos x$

$$\frac{du}{dx} = e^x$$

$$v = \sin x$$

$$\begin{aligned} \int e^x \cos x dx &= e^x \sin x - \int \sin x \cdot e^x dx \\ &= e^x \sin x - \int e^x \sin x dx \end{aligned}$$

$$\int e^x \sin x dx = -e^x \cos x + e^x \sin x - \int e^x \sin x dx$$

adding $\int e^x \sin x dx$ to both sides

$$2 \int e^x \sin x dx = -e^x \cos x + e^x \sin x$$

$$2 \int e^x \sin x dx = e^x (\sin x - \cos x)$$

$$\int e^x \sin x dx = \frac{e^x}{2} (\sin x - \cos x) + C$$

$$(2) \int 2x^2 \ln x \, dx$$

$$\boxed{\int u \, dv = uv - \int v \, du}$$

$$\text{Let } u = \ln x \rightarrow du = \frac{1}{x} \, dx$$

$$\text{Let } dv = 2x^2 \rightarrow v = \frac{2x^3}{3}$$

$$\int 2x^2 \ln(x) \, dx = \ln(x) \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3} \cdot \frac{1}{x} \, dx$$

$$~~\frac{2x^3}{3}~~ = \frac{2x^3}{3} \ln(x) - \frac{2}{3} \int x^2 \, dx$$

$$= \frac{2x^3}{3} \ln(x) - \frac{2}{3} \cdot \frac{x^3}{3}$$

$$= \frac{x^3}{3} \cdot \ln(x) - \frac{2x^3}{9} + C$$

$$(3) \int x^2 \sin x \, dx$$

$$\boxed{\int u \, dv = uv - \int v \, du}$$

$$\text{Let } u = x^2$$

$$du = 2x$$

$$\text{and } dv = \sin x$$

$$v = -\cos x$$

$$\int x^2 \sin x \, dx = x^2(-\cos x) - \int (-\cos x) 2x \, dx$$

$$= -x^2 \cos x + 2 \int x \cos x \, dx$$

$$\int x \cos x \, dx$$

$$\text{Let } u = x$$

$$du = 1$$

$$\text{and } dv = \cos x$$

$$v = \sin x$$

$$\begin{aligned} \int x \cos x \, dx &= x \sin x - \int \sin x \, dx \\ &= x \sin x - (-\cos x) \\ &= \boxed{x \sin x + \cos x} \end{aligned}$$

$$\begin{aligned} \therefore \int x^2 \sin x \, dx &= -x^2 \cos x + 2[x \sin x + \cos x] + C \\ &= \underline{\underline{2x \sin x + 2\cos x - x^2 \cos x + C}} \end{aligned}$$

$$(4) \int x \cos x \, dx$$

$$\boxed{\int u \, dv = uv - \int v \, du}$$

$$\text{let } u = x \quad \text{and } dv = \cos(x)$$

$$du = dx \quad v = \sin(x)$$

$$\begin{aligned} \therefore \int x \cos(x) \, dx &= \int \sin(x) \, dx \cdot x - \sin(x) - \int \sin(x) \, dx \\ &= x \sin(x) + \int -\sin(x) \, dx \\ &= \underline{\underline{x \sin(x) + \cos(x) + C}} \end{aligned}$$