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Elect / Elect Engr

19/ENG04/004

MAT 104

Serial No : 12

(1.) $y = \sin\left(\frac{6}{x^2}\right)$; $y = \sin(6x^{-2})$

$$y + \Delta y = \sin[6(x+\Delta x)^{-2}]$$

$$\Delta y = \sin[6(x+\Delta x)^{-2}] - \sin(6x^{-2})$$

$$A \quad - \quad B$$

$\sin A - \sin B$; $A = 6(x+\Delta x)^{-2}$: $B = 6x^{-2}$

$$= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\Delta y = 2\cos\left(\frac{6(x+\Delta x)^{-2} + 6x^{-2}}{2}\right)\sin\left(\frac{6(x+\Delta x)^{-2} - 6x^{-2}}{2}\right)$$

$$\Delta y = 2\cos\left(\frac{12x^{-2} + 6\Delta x^{-2}}{2}\right)\sin\left(\frac{6\Delta x^{-2}}{2}\right)$$

divide both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\left(\frac{12x^{-2} + 6\Delta x^{-2}}{2}\right)\sin\left(\frac{6\Delta x^{-2}}{2}\right)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\cancel{2}\cos\left(\frac{12x^{-2} + 6\Delta x^{-2}}{2}\right)\sin\left(\frac{6\Delta x^{-2}}{2}\right) \times 1/\cancel{2}}{\Delta x/2}$$

$$\frac{\Delta y}{\Delta x} = \frac{\cos\left(\frac{12x^{-2}+6\Delta x^{-2}}{2}\right)}{1} \cdot \frac{\sin\left(\frac{6\Delta x^{-2}}{2}\right)}{\Delta x/2}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cos\left(\frac{12x^{-2}+6\Delta x^{-2}}{2}\right)}{1} \cdot \lim_{\Delta x \to 0} \frac{\sin\left(\frac{6\Delta x^{-2}}{2}\right)}{\Delta x/2}$$

$$\lim_{\Delta x \to 0} \frac{\cos\left(\frac{12x^{-2}+6\Delta x^{-2}}{2}\right)}{1} = \cos\left(\frac{12x^{-2}}{2}\right)$$

$$= \cos(6x^{-2}),$$

$$\lim_{\Delta x \to 0} \frac{\sin\left(\frac{6\Delta x^{-2}}{2}\right)}{\Delta x/2} = 1$$

$$\frac{dy}{dx} = \cos(6x^{-2}) \cdot 1$$

$$\frac{dy}{dx} = \cos(6x^{-2}) \text{ or } \cos\left(\frac{6}{x^2}\right),$$

(2.) Area under the curve = ?

Parametric equations $x = 4t^3 - t^2$ at $t = 1$

$y = t^4 - 2t^2$ $t = 3$

$$\int_a^b y \, dx$$

$$\int_1^3 t^4 - 2t^2 \, dx$$

$$x = 4t^3 - t^2$$

$$dx/dt = 12t^2 - 2t$$

$$dx = 12t^2 - 2t \, dt$$

$$\int_1^3 t^4 - 2t^2 (12t^2 - 2t \, dt)$$

$$\int_1^3 12t^6 - 2t^5 - 24t^4 + 4t^3 \, dt$$

$$\left[\frac{12(3)^7}{7} - \frac{2(3)^6}{6} - \frac{24(3)^5}{5} + \frac{4(3)^4}{4}\right] -$$

$$\left[\frac{12(1)^7}{7} - \frac{2(1)^6}{6} - \frac{24(1)^5}{5} + \frac{4(1)^4}{4}\right]$$

$$\cancel{= \frac{36^7}{7} - \frac{6^6}{6} - \frac{72^5}{5} + \frac{12^3}{4}} -$$

$(3749.14 - 243 - 1166.4 + 81) -$
$(1.71 - 0.33 - 4.8 + 1)$

$= 2420.74 - (-2.42)$

$= 2420.74 + 2.42$

$= 2423.16$ square units

(3.) Find dy/dx

If $x = 4t^3 - t^2$

$y = t^4 + 2t^2$

~~Fin~~ ~~dy/dx~~ $= y$

$y = t^4 + 2t^2 \; dx$

$x = 4t^3 - t^2$

$dx/dt = \cancel{\ } 12t^2 - 2t$

$dx = 12t^2 - 2t \; dt$

$y = t^4 + 2t^2 (12t^2 - 2t \; dt)$

$y = 12t^6 - 2t^5 + 24t^4 - 4t^3 \; dt$

$dy/dx = 72t^5 - 10t^4 + 96t^3 - 12t^2$