A **linear transformation**, T:U→VT:U→V, is a function that carries elements of the vector space UU (called the **domain**) to the vector space VV (called the **codomain**), and which has two additional properties

1. T(u1+u2)=T(u1)+T(u2)T(u1+u2)=T(u1)+T(u2) for all u1,u2∈Uu1,u2∈U
2. T(αu)=αT(u)T(αu)=αT(u) for all u∈Uu∈U and all α∈C

$$A = \begin{pmatrix} 0 & 1 & 3 \\ -1 & 0 & 2 \\ 4 & -2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 4 & 3 & 0 \\ 6 & 1 & -1 \\ 5 & 2 & -4 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 4 & 2 & 1 \end{pmatrix} \qquad E = \begin{pmatrix} 3 & 3 & 0 \\ 2 & 1 & -1 \\ 1 & 3 & -1 \end{pmatrix}$$

i. LINEAR TRANSFORMATION OF A , IF VEXTOR x = (a,b,c)

$$A = \begin{pmatrix} 0 & 1 & 3 \\ -1 & 0 & 2 \\ 4 & -2 & 1 \end{pmatrix} \qquad x = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$T(x) = a \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix} + b \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + c \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

4 2 1

3 1 -1

2 0 + 1

$$\begin{pmatrix} 0 \\ -a \\ 4a \end{pmatrix} + \begin{pmatrix} b \\ 0 \\ -2b \end{pmatrix} + \begin{pmatrix} 3c \\ 2c \\ c \end{pmatrix}$$

T(x) =

Hence the transformation of $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ gives; $\begin{pmatrix} b + 3c \\ -a + 2c \\ 4a - 2b + c \end{pmatrix}$

ii. **LINEAR TRANSFORMATION OF B , IF VEXTOR x = (a,b,c)**

B = x = $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

0 + b + 3c

-a + 0 + 2c

4a + 2b + c

3a + b - c

2a + 0 + c

$$T(x) = a\begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} + b\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + c\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 4a \\ 3a \\ 2a \end{pmatrix} + \begin{pmatrix} 2b \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} c \\ -c \\ c \end{pmatrix}$$

T(x) =

Hence the transformation of $\begin{pmatrix}a\\b\\c\end{pmatrix}$ gives; $\begin{pmatrix}4a + 2b + c\\3a + b - c\\2a + c\end{pmatrix}$

iii. **LINEAR TRANSFORMATION OF C , IF VEXTOR x = (a,b,c)**

C = $\begin{pmatrix}4&3&0\\6&1&-1\\5&2&-4\end{pmatrix}$ x = $\begin{pmatrix}a\\b\\c\end{pmatrix}$

$$T(x) = a\begin{pmatrix}4\\6\\5\end{pmatrix} + b\begin{pmatrix}3\\1\\2\end{pmatrix} + c\begin{pmatrix}0\\-1\\-4\end{pmatrix}$$

$$\begin{pmatrix}4a\\6a\\5a\end{pmatrix} + \begin{pmatrix}3b\\b\\2b\end{pmatrix} + \begin{pmatrix}0\\-c\\-4c\end{pmatrix}$$

$$T(x) = \begin{pmatrix}4a + 3b + 0\\6a + b - c\\5a + 2b - 4c\end{pmatrix}$$

Hence the transformation of $\begin{pmatrix}a\\b\\c\end{pmatrix}$ gives; $\begin{pmatrix}4a + 3b\\6a + b - c\\5a + 2b - 4c\end{pmatrix}$

iv. **LINEAR TRANSFORMATION OF D , IF VEXTOR x = (a,b,c)**

D = $\begin{pmatrix}0&1&3\\1&0&2\\4&2&1\end{pmatrix}$ x = $\begin{pmatrix}a\\b\\c\end{pmatrix}$

$$T(x) = a\begin{pmatrix}0\\1\\4\end{pmatrix} + b\begin{pmatrix}1\\0\\2\end{pmatrix} + c\begin{pmatrix}3\\2\\1\end{pmatrix}$$

$$\begin{matrix}3 & 3 & 0\\2 & 1 & -1\\1 & 3 & -1\end{matrix}$$

$$\begin{pmatrix}0\\a\\4a\end{pmatrix} + \begin{pmatrix}b\\0\\2b\end{pmatrix} + \begin{pmatrix}3c\\2c\\c\end{pmatrix}$$

$$T(x) = \begin{pmatrix} \\ \\ \end{pmatrix}$$

Hence the transformation of $\begin{pmatrix}a\\b\\c\end{pmatrix}$ gives; $\begin{pmatrix}b + 3c\\a + 2c\\4a + 2b + c\end{pmatrix}$

v. LINEAR TRANSFORMATION OF E , IF VEXTOR x = (a,b,c)

$E = \begin{pmatrix} \\ \\ \end{pmatrix}$ $\quad x = \begin{pmatrix}a\\b\\c\end{pmatrix}$

$$\begin{matrix}0 + b + 3c\\a + 2c\\4a + 2b + c\end{matrix}$$

$$T(x) = a\begin{pmatrix}3\\2\\1\end{pmatrix} + b\begin{pmatrix}3\\1\\3\end{pmatrix} + c\begin{pmatrix}0\\-1\\-1\end{pmatrix}$$

$$\begin{pmatrix}3a\\2a\\a\end{pmatrix} + \begin{pmatrix}3b\\b\\3b\end{pmatrix} + \begin{pmatrix}0\\-c\\-c\end{pmatrix}$$

$$T(x) = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

Hence the transformation of $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ gives; $\begin{pmatrix} 3a + 3b + 0 \\ 2a + b - c \\ a - 3b - c \end{pmatrix}$

$$\begin{matrix} 3a + 3b + 0 \\ 2a + b - c \\ a - 3b - c \end{matrix}$$