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Computer Engineering

19/ENG.02/006

Maths 104 assignment

2) Find the area under the curve given parametric equations

$$x = 4t^3 - 2 \text{ and } y = t^4 + 2t^2 \text{ at } t = 1 \text{ and } t = 3:$$

Solution:

Let A represent the area then $A = \int_a^b y dx$

where a and b are the limit values for the variable.

$$A = \int_a^b y dx$$

$$x = 4t^3 - 2$$

$$\frac{dx}{dt} = 12t^2 - 2t$$

$$dx = (12t^2 - 2t) dt$$

$$A = \int_1^3 (t^4 + 2t^2) (12t^2 - 2t) dt$$

$$A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3) dt$$

$$A = \left[\frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4} \right]_1^3$$

$$A = \left[\frac{12t^7}{7} - \frac{t^6}{3} + \frac{24t^5}{5} - t^4 \right]_1^3$$

$$A = \left(\frac{12(3)^7}{7} - \frac{(3)^6}{3} + \frac{24(3)^5}{5} - (3)^4 \right) -$$

$$\left(\frac{12(10)^7}{7} - \frac{(10)^6}{3} + \frac{24(10)^5}{5} - (10)^4 \right)$$

$$A = \frac{160704}{35} - \frac{544}{105}$$

$$A = \frac{451568}{105}$$

$$A = 4586.4$$

37) If $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$ - find $\frac{dy}{dx}$.

Solution -

$$x = 4t^3 - t^2$$

$$y = t^4 + 2t^2$$

$$\frac{dx}{dt} = 12t^2 - 2t$$

$$\frac{dy}{dt} = 4t^3 + 4t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 4t^3 + 4t, \quad \frac{dt}{dx} = \frac{1}{\frac{dx}{dt}}$$

$$\frac{dt}{dx} = \frac{1}{12t^2 - 2t}$$

$$\frac{dy}{dx} = \frac{4t^3 + 4t}{1} \times \frac{1}{12t^2 - 2t} = \frac{4t^3 + 4t}{12t^2 - 2t}$$

$$\frac{dy}{dx} = \frac{2t(2t^2 + 2)}{2t(6t - 1)}$$

$$\frac{dy}{dx} = \frac{2t^2 + 2}{6t - 1}$$

$$\therefore \frac{dy}{dx} = \frac{2t^2 + 2}{6t - 1}$$