NAME: Opata Clinton Chukwudubem

Date: 15th ~~Of~~ May 2020

Dept: Aeronautical Engineering

Matric NO: 19/ENG09/019

General Mathematics II, MAT104

Assignment for Dr. OYELAMI's Group

Questions and Answers.

Find the Integral of the following

1) $\int e^x \sin x \, dx$

solution

Using Integration by parts

$uv - \int v\,du$, According to LIATE

$u = \sin x$ and, $du = \cos x\,dx$, $dv = e^x\,dx$

$v = e^x$

$\therefore \int e^x \sin x\,dx = uv - \int v\,du = e^x \sin x$

$- \int e^x \cos x\,dx$

$\therefore$ consider $\int e^x \cos x\,dx$, $u = \cos x$, $du = -\sin x\,dx$

$v = e^x$, $dv = e^x dx$, substituting into the integration

$\therefore \int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx$

from $e^x \sin x - \int e^x \cos x \, dx$

$\int e^x \sin x \, dx = e^x \sin x - [e^x \cos x + \int e^x \sin x \, dx]$

$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

collecting like terms.

$\int e^x \sin x \, dx + \int e^x \sin x \, dx = e^x \sin x - e^x \cos x$

$2\int e^x \sin x \, dx = e^x \sin x + e^x \cos x$

Divide both sides by the coefficient of $\int e^x \sin x \, dx$

$\therefore \frac{2\int e^x \sin x \, dx}{2} = \frac{e^x \sin x}{2} + \frac{e^x \cos x}{2}$

$\therefore \int e^x \sin x \, dx = \frac{e^x \sin x}{2} + \frac{e^x \cos x}{2} + C$

ans.

2) $2x^2 \ln x \, dx$

solution

$\int 2x^2 \ln x \, dx$ using integration by parts.

$u = \ln x, \; dv = 2x^2\,dx$

$du = \frac{dx}{x}, \; v = \int 2x^2\,dx = \frac{2x^3}{3}$

$\therefore \int 2x^2 \ln x\,dx = \frac{2}{3}x^3 \ln x - \int \frac{2}{3} x^3 \frac{dx}{\cancel{x}}$

$\int 2x^2 \ln x\,dx = \frac{2}{3}x^3 \ln x - \int \frac{2}{3} x^2\,dx$

$\int 2x^2 \ln x\,dx = \frac{2}{3}x^3 \ln x - \frac{2}{3}\int x^2\,dx$

$\int 2x^2 \ln x\,dx = \frac{2}{3}x^3 \ln x - \frac{2}{3}\left[\frac{x^3}{3}\right] = \frac{2}{9}x^2 \ln x + \frac{2}{9}x^3$

$\therefore \int 2x^2 \ln x\,dx = \frac{2}{3}x^3 \ln x - \frac{2}{9}x^3 + C$

ans

3) $x^2 \sin x\,dx$

Solution

$\int x^2 \sin x\,dx$ using integration by parts

$u = x^2$ and $dv = \sin x\,dx$, $\frac{du}{dx} = 2x \therefore du = 2x\,dx$

$v = -\cos x \; \{\int \sin x\,dx\}$

$\therefore \int x^2 \sin x\,dx = -x^2 \cos x - \int -\cos x \, 2x\,dx$ --(1)

Arranging it by simplifying,

$\int x^2 \sin x\,dx = -x^2 \cos x + \int 2x \cos x\,dx$

But consider $\int 2x\cos x\,dx$, $u = 2x$, $dv = \cos x\,dx$

$\frac{du}{dx} = 2$, $du = 2\,dx$, $v = \sin x$

$\therefore \int 2x\cos x\,dx = 2x\sin x - 2\int \sin x\,dx$

$\int 2x\cos x\,dx = 2x\sin x + 2\cos x$

substituting into (i)

$\int x^2 \sin x\,dx = -x^2\cos x + \int 2x\cos x\,dx$

$\therefore \int x^2\sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + C$ ans.

4) $x\cos x\,dx$

solution

$u = x$, $dv = \cos x$, $du = dx$, $V = \sin x$

$\int x\cos x\,dx = uv - \int v\,du = x\sin x - \int \sin x\,dx$

$\int x\cos x\,dx = x\sin x - (-\cos x)$

$\int x\cos x\,dx = x\sin x + \cos x + C$

ans.