NAME: TOWURU Jesutifunmi Nissi
DEPT: COMPUTER ENGINEERING
MATRIC NUMBER: 18/ENG02/095
Course code & Title:

Question:

1. differentiate $y = \sin(6/x^2)$ from the first principle.
2. Find the area under the curve, given parametric equations $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$, at $t = 1$ and $t = 3$.
3. If $x = 4^3 - t^2$ and $y = t^4 + 2t^2$, find dy/dx.

Solution:

1. differentiate $y = \sin(6/x^2)$ from the first principle.

Apply the constant multiple rule
$\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}(f(x))$ with $c = y$ and

$$f(x) = \sin\left(6/x^2\right)$$

$$\frac{d}{dx}\left(y\sin\left(6/x^2\right)\right) = y\left(\frac{d}{dx}\right)\left(\sin\left(6/x^2\right)\right)$$

$$\frac{d}{dx}\left[\sin\left(\frac{6}{x^2}\right)\right]$$

$$= \cos\left(\frac{6}{x^2}\right) \cdot \frac{d}{dx}\left[6/x^2\right]$$

The derivative of sine is $\frac{d}{du}(\sin(u)) = \cos(u)$:

$$y\frac{d}{du}(\sin(u))\frac{du}{dx}\left(6/x^2\right) = y\cos(u)\frac{d}{dx}\left(6/x^2\right)$$

$$= \cos\left(6/x^2\right) \cdot 6 \cdot \frac{d}{dx}\left[1/x^2\right]$$

$$= 6\cos\left(6/x^2\right)(-2)x^{-3}$$

$$= -\frac{12\cos(6/x^2)}{x^3}$$

3 If $x = 4t^3 - t^2$ and
$y = t^4 + 2t^2$ find dy/dx

Solution

formula - $\frac{dy}{dx}(x) = \frac{\frac{dy(t)}{dt}}{\frac{dx(t)}{dt}}$

$$\frac{dy(t)}{dt} = \frac{d}{dt}\left(t^4 + 2t^2\right)$$

$$\frac{d(t^4)}{dt} + \frac{d(2t^2)}{dt}$$

$$\frac{d(t^4)}{dt} = 4 \cdot t^{4-1}$$

$$= 4 \cdot t^3 + \frac{d(2 \cdot t^2)}{dt}$$

$$= 4 \cdot t^3 + 2 \frac{(t^2)}{dt}$$

$$= 4 \cdot t^3 + 2 \cdot 2t$$

we now have

$$\frac{dy}{dt}(t) = 4 \cdot t^3 + 4 \cdot t$$

we calculate the derivative

$$\frac{dx(t)}{dt}:$$

$$\frac{d(4 \cdot t^3)}{dt} + \frac{d(-t^2)}{dt}$$

$\frac{d(t^3)}{dt} = 3t^{3-1}$

$= 4 \cdot 3 \cdot t^{3-1} + \frac{d(-t^2)}{dt}$

find similar terms $3-1=2$:

$4 \cdot 3 \cdot t^2 + \frac{d(-t^2)}{dt}$

find similar terms: $4 \cdot 3 = 12$:

$12 \cdot t^2 + \frac{d(-t^2)}{dt}$

$= 12 \cdot t^2 - \frac{d(t^2)}{dt}$

$= \frac{d(t^2)}{dt} = 2 \cdot t^{2-1}$

$= 12 \cdot t^2 - 2 \cdot t^{2-1}$

$= 12 \cdot t^2 - 2 \cdot t$

we have

$\frac{dx(t)}{dt} = 12 \cdot t^2 - 2 \cdot t$

substitute into initial formula.

$\frac{dy(x)}{dx} = \frac{\frac{dy(t)}{dt}}{\frac{dx(t)}{dt}} = \frac{4 \cdot t^3 + 4 \cdot t}{12 \cdot t^2 - 2 \cdot t}$

$= \frac{4t^3 + 4t}{12t^2 - 2t}$ // Answer

(3) Find the area under the curve, given parametric equations $x = 4t^3 - t^2$ and $y = t^4 + 2t^2$ at $t=1, t=3$

$x = 4t^3 - t^2$

$y = t^4 + 2t^2$ at

$A = \int_a^b y\,dx \quad y = t^4 + 2t^2$

$A = \int_1^3 t^4 + 2t^2$

Given $x = 4t^3 - t^2$

$dx/dt = 12t^2 - 2t$

$dx = 12t^2 - 2t \cdot dt$

$A = \int^3 (t^4 + 2t^2) \times (12t^2 - 2t)$

$A = \int_1^3 (12t^2 - 2t) + 2t^2(12t^2 - 2t)$

$12t^6 - 2t^5 + 24t^4 - 4t^3$

$A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3)\,dt$

$A = \int_1^3 \frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4}$

$A = \left[\frac{12(3)^7}{7} - \frac{2(3)^6}{6} \times \frac{24(3)^5}{5} - \frac{4(3)^4}{4}\right] -$

$\left[\frac{12}{7} - 2/6 - 16/5 - 4/4\right]$

$A = \left[\frac{26244}{7} - 243 + 1166.4 - 81\right] - \left[\frac{544}{105}\right]$

$= A = 2420.74 - 5.1809$

$A = 2815.5591$

$A \simeq 2815.6$ Square units//