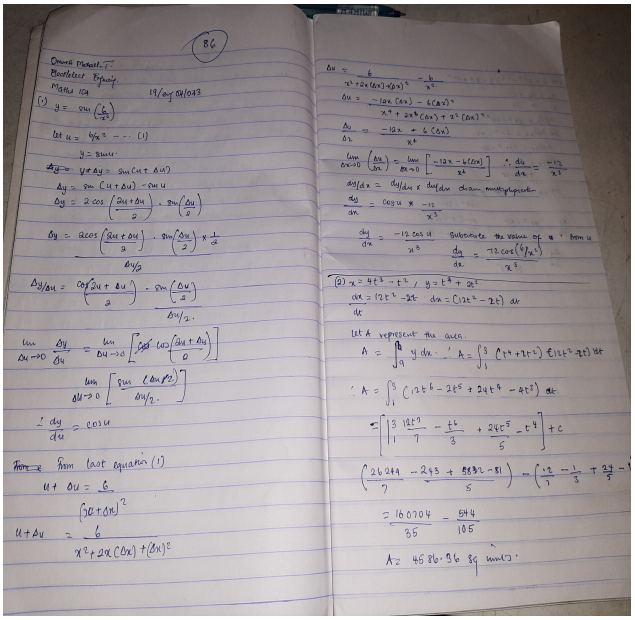


86

Quick Method - 7
Bhattacharya Exam.
Maths 10A
19/09/2023

(1) $y = \sin\left(\frac{6}{x^2}\right)$

let $u = 6/x^2$ ... (1)

$y = \sin u$.

~~If~~ $y + \Delta y = \sin(u + \Delta u)$

$\Delta y = \sin(u + \Delta u) - \sin u$

$\Delta y = 2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)$

$\frac{\Delta y}{\Delta u/2} = \frac{2\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right) \times \frac{1}{2}}{\Delta u/2}$

$\frac{\Delta y}{\Delta u} = \frac{\cos\left(\frac{2u + \Delta u}{2}\right) \cdot \sin\left(\frac{\Delta u}{2}\right)}{\Delta u/2.}$

$\lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} = \lim_{\Delta u \to 0} \left[\cos\left(\frac{2u + \Delta u}{2}\right)\right]$

$\lim_{\Delta u \to 0} \left[\frac{\sin(\Delta u/2)}{\Delta u/2.}\right]$

$\frac{dy}{du} = \cos u$

From ~~2~~ from last equation (1)

$u + \Delta u = \frac{6}{(x + \Delta x)^2}$

$u + \Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2}$

$\Delta u = \frac{6}{x^2 + 2x(\Delta x) + (\Delta x)^2} - \frac{6}{x^2}$

$\Delta u = \frac{-12x(\Delta x) - 6(\Delta x)^2}{x^4 + 2x^3(\Delta x) + x^2(\Delta x)^2}$

$\frac{\Delta u}{\Delta x} = \frac{-12x + 6(\Delta x)}{x^4}$

$\lim_{\Delta x \to 0}\left(\frac{\Delta u}{\Delta x}\right) = \lim_{\Delta x \to 0}\left[\frac{-12x - 6(\Delta x)}{x^4}\right]$ $\therefore \frac{du}{dx} = \frac{-12}{x^3}$

$dy/dx = dy/du \times du/dx$ chain multiplication

$\frac{dy}{dx} = \cos u \times \frac{-12}{x^3}$

$\frac{dy}{dx} = \frac{-12\cos u}{x^3}$ Substitute the value of u from u

$\frac{dy}{dx} = \frac{-12\cos\left(\frac{6}{x^2}\right)}{x^3}$

(2) $x = 4t^3 - t^2$; $y = t^4 + 2t^2$

$\frac{dx}{dt} = 12t^2 - 2t$ $dx = (12t^2 - 2t)\,dt$

Let A represent the area.

$A = \int_a^b y\,dx$ $\therefore A = \int_1^3 (t^4 + 2t^2)(12t^2 - 2t)\,dt$

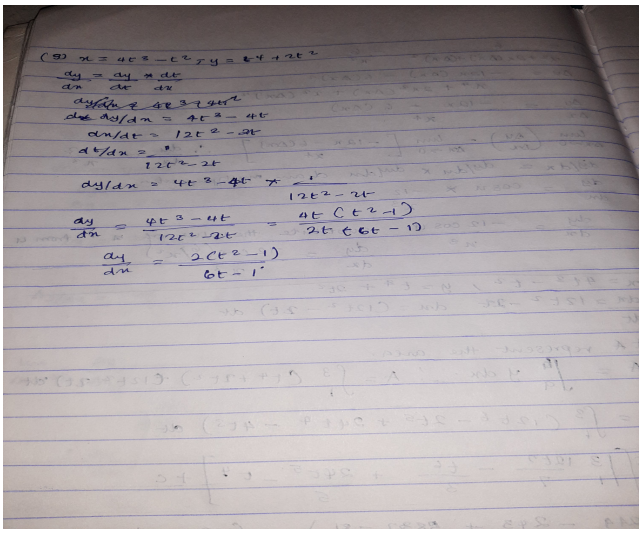
$\therefore A = \int_1^3 (12t^6 - 2t^5 + 24t^4 - 4t^3)\,dt$

$= \left[\frac{12t^7}{7} - \frac{t^6}{3} + \frac{24t^5}{5} - t^4\right] + C$

$\left(\frac{26244}{7} - 243 + \frac{8832}{5} - 81\right) - \left(\frac{12}{7} - \frac{1}{3} + \frac{24}{5} - 1\right)$

$= \frac{160704}{35} = \frac{544}{105}$

$A = 4586.96$ sq units.



(3) $x = 4t^3 - t^2$; $y = t^4 + 2t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$dy/dx = 4t^3 + 4t^2$

$dy/dx = 4t^3 - 4t$

$dx/dt = 12t^2 - 2t$

$dt/dx = \frac{1}{12t^2 - 2t}$

$dy/dx = 4t^3 - 4t \times \frac{1}{12t^2 - 2t}$

$\frac{dy}{dx} = \frac{4t^3 - 4t}{12t^2 - 2t} = \frac{4t(t^2 - 1)}{2t(6t - 1)}$

$\frac{dy}{dx} = \frac{2(t^2 - 1)}{6t - 1}$