1. A vector space over a field is a set that is closed under finite vector addition and scalar multiplication for V to be a vector space.
2. A=(1,1,1)

B=(1,2,3)

C=(1,5,8)

Solution

α +β +δ

 1 1 1 a

 1 2 5 = b

 1 3 8 c

=α…….1

α =b………2

α =c………..3

α=a-β-δ=c………..4

 sub 4 into 2 and 3

 (α) +β+δ=b

 a-β-δ+β+δ=b

 a+β+4δ=b

 a+β+4δ=b…………5

a+2β+7δ=b…………6

-β-3δ=b-c

 -β=b-c+3δ

Β=-b+c-3δ

 (a-β-δ)+3β+8δ=c

 a-β-δ+3β+8δ=c

 a+2β+7δ=c

 sub β into 5

a+(-b+c-3δ)+4δ=b

a-b+c-3δ+4δ=b

a-b+c+δ=b

δ=b-a+b-c

δ=2b-a-c

Sub δ into β

Β=-b+c-3(2b-a-c)

Β=-b+c-6b+3a+3c

Β=-7b+3a+4c

Sub β and 5 into 4

α=a-(-7b+3a+4c)-(2b-a-c)

=a+7b-3a-4c-2b+a+c

=-a+5b-3c

1. P=(1,2,3)

 Q=(3,2,1)

 R=(0,0,1)

SOLUTION

First check if its linear or not

α +β +δ

 1 3 0 0

 2 2 0 = 0

 3 1 1 0

α+3β=0……..1

2α+2β=0…….2

3α+β+δ=0……3

From equation 1

α=-3β……..4

sub 4 into 2

2(-3β)+2β=0

-6β+2β=0

-4β=0

β=0

subβ into 4

α=-3(0)

α=0

sub α andβ into 3

3(0)+0+δ=0

0+0+δ=0

δ=0

α=0, β=0 and δ=0

hence the vectors are linearly dependent

 Spanning Sets

α +β +δ

 1 3 0 0

 2 2 0 = 0

 3 1 1 0

α+3β=0……..1

2α+2β=0…….2

3α+β+δ=0……3

From equation 1

α=a-3β………4

sub 4 into 2

2(a-3β)+2β=b

2a-6β+2β=b

2a-4β =b

4β=2a-b

Β=2a-b

 4

Sub β into 4

Α=a-3

 2a-b

 4

α=a-6a+3b

 4

α=4a-6a+3b

 4

α=-2a+3b

 4

Sub α and β into 3

3

 2a-b +2a-b +δ=c

 4 4

Multiply through by 4

3(-2a+3b)+(2a-b)+4δ=4c

-6a+9b+2a-b+4δ=4c

-4a+8b+4δ=4c

Divide through by 4

-a+2b+δ=c

δ=c+a-2b

Since the vectors are linearly independent and spans of R3 sothe vectors are basis of R3