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MATRIC NO 19/ENG02/1015
COURSE MAT 104

① Differentiate $y = \sin(6/x^2)$ from first principle

Soln

$y = \sin(6x^{-2})$

$y + \Delta y = \sin(6x^{-2} + 6\Delta x^{-2})$

$\Delta y = \sin(6x^{-2} + 6\Delta x^{-2}) - \sin 6x^{-2}$

$\frac{2\cos(A+B)}{2} \cdot \sin\frac{(A-B)}{2}$

$2\cos\left(\frac{6x^{-2} + 6\Delta x^{-2} + 6x^{-2}}{2}\right) \sin\left(\frac{6x^{-2} + 6\Delta x^{-2} - 6x^{-2}}{2}\right)$

$= 2\cos\left(\frac{12x^{-2} + 6\Delta x^{-2}}{2}\right) \sin\left(\frac{6\Delta x^{-2}}{2}\right)$

divide both sides by Δx

$\frac{\Delta y}{\Delta x} = \frac{2\cos\left(\frac{12x^{-2} + 6\Delta x^{-2}}{2}\right) \sin\left(\frac{6\Delta x^{-2}}{2}\right)}{\Delta x}$

$\lim_{\Delta x \to 0} = \cos\left(\frac{12x^{-2} + 6\Delta x^{-2}}{2}\right) \frac{\sin\left(\frac{6\Delta x^{-2}}{2}\right)}{\frac{\Delta x}{2}}$

$\lim_{\Delta x \to 0} = \cos\left(\frac{12x^{-2} + 6(0)^{-2}}{2}\right) \frac{\sin\frac{6\Delta x^{-2}}{2}}{\frac{\Delta x}{2}} \to 1$

$\frac{\Delta y}{\Delta x} = \cos\frac{6x^{-2}}{2}$

$= \cos 6x^{-2}$

② Find the area under the curve given parametric equations $x = 4t^3 - t^2$ & $y = t^4 + 2t^2$ at $t = 1$ & $t = 3$

Soln

$A = \int_a^b y\,dx$

$A = \int_1^3 t^4 + 2t^2\,dx$

Given $x = 4t^3 - t^2$

$\frac{dx}{dt} = 12t^2 - 2t$

$dx = (12t^2 - 2t)\,dt$

$A = \int_1^3 t^4 + 2t^2 (12t^2 - 2t)\,dt$

$$A = \int_1^3 12t^6 - 2t^5 + 24t^4 - 4t^3 \, dt$$

$$\left[\frac{12t^7}{7} - \frac{2t^6}{6} + \frac{24t^5}{5} - \frac{4t^4}{4}\right]$$

$$= \left[\frac{12(3)^7}{7} - \frac{2(3)^6}{6} + \frac{24(3)^5}{5} - \frac{4(3)^4}{4}\right] - \left[\frac{12(1)^7}{7} - \frac{2(1)^6}{6} + \frac{24(1)^5}{5} - \frac{4(1)^4}{4}\right]$$

$$\left[\frac{26244}{7} - 1053\right] - \left[\frac{12}{7} - \frac{2}{6} + \frac{24}{5} - 1\right]$$

$$\left[\frac{26244 - 7371}{7}\right] - \left[\frac{544}{105}\right]$$

$$\left[\frac{18873}{7}\right] - \left[\frac{544}{105}\right]$$

$$= 2690.96$$

3) If $x = 4t^3 - t^2$ & $y = t^4 + 2t^2$ find dy/dx

$$\frac{dx}{dt} = 12t^2 - 2t \qquad \frac{dy}{dt} = 4t^3 + 4t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$= \frac{4t^3 + 4t}{12t^2 - 2t}$$