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18/SCI01/030

MAT 204 ASSIGNMENT

Define a vector space

A vector space over a real field F is a set that is closed under finite vector addition and scalar multiplication.

Show that the vectors A =(1, 1, 1) , B =(1, 2, 3), C =(1, 5, 8) spans Rᵌ .

 Solution

 αA + βB + δC

α 1 + β 1 + δ 1 a

 1 2 5 = b

 1 3 8 c

 α + β + δ = a – (1)

 α + 2β + 5δ = b – (2)

 α + 3β + 8δ = c – (3)

from eqn (1)

α = a - β - δ – (4)

put (4) in (2) and (3)

α + 2β + 5δ = b

(a - β - δ) + 2β + 5δ = b

a - β - δ + 2β + 5δ = b

a + β + 4δ = b - a –(5)

α + 3β + 8δ = c

(a - β - δ) + 3β + 8δ = c

 a - β - δ + 3β + 8δ = c

 a + 2β + 7δ = c - a –(6)

combine equation (5) and (6)

 β + 4δ = b - a – (5)

 -

 2β + 7δ = c - a – (6)

 Multiply equation (5) by 2 and equation (6) by 1

= 2β + 8δ = 2b – 2a

 - 2β + 7δ = c – a

 δ = (2b – 2a) – (c – a)

 δ = 2b – 2a – c + a

 δ = 2b – a – c

 δ = -a + 2b – c

From equation (5)

β + 4δ = b - a

Substitute the value of δ

= β + 4( -a + 2b –c) = b – a

 β - 4a + 8b –4c = b – a

 β = 4a - 8b + 4c = b – a

 β = 3a - 7b + 4c

From equation (4)

α = a - β - δ

α = a - (3a - 7b + 4c) - (-a + 2b – c)

α = a - 3a + 7b - 4c +a - 2b + c

 = -a + 5b -3c

Are the vectors A = (1, 2, 3), Q = (3, 2, 1), R = (0, 0, 1) a basis for Rᵌ?

 Solution

First check for linear dependency

 αP + βQ + δR

α 1 + β 3 + δ 0 0

 2 2 0 = 0

 3 1 1 0

 α + 3β = 0 – (1)

 2α + 2β = 0 – (2)

 3α + β + δ = 0 – (3)

From eqn (1)

α + β = 0

Substitute eqn (1) in (2) and (3)

 2α + 2β = 0

 2(-3β) + 2β = 0

 -6β + 2β = 0

 -4β = 0

 β = 0

Substitute values of β into (2)

α + 3β = 0

α + 0 = 0

α = 0

 Substitute α and β into equation (3)

3α + β + δ = 0

3(0) + 0 + δ = 0

δ = 0

 α + 3β = p – (1)

 2α + 2β = q – (2)

 3α + β + δ = r – (3)

Combine equations (1) and (2)

 α + 3β = p x1

 2α + 2β = q x2

 Multiply equations (1) by 2 and (2) by 1

 2α + 6β = 2p

 - 2α + 2β = q

 4β = 2p – q

 4 4

 β = 2p – q

 4

 Substitute value of β into (1)

 α + 3β = p

 α + 3 2p – q = p

 4

 α + 6p –3q = p

 4

 α = p - 6p –3q

 1 4

α = 4p – (6p – 3q)

 4

α = 4p – 6p + 3q

 4

α = –2p + 3q

 4

In equation (2)

3α + β + δ = r

δ = r - 3α – β

δ = r - 3 -2p + 3q - 2p – q

 1 4 4

δ = 4r – 3 (-2p + 3q) – (2p – q)

 4

δ = 4r + 6p - 9q – 2p + q

 4

δ = 4r + 4p - 8q

 4

δ = 4p - 8q + 4r

 4