19/ENG05/060 Dr Oyetomi

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MAT104

Mechatronics Engineering

(1) $\int e^{2} \sin x \, dx$

Solution

$u = \sin x \qquad dv = e^{x}$

$\frac{du}{dx} = \cos x \qquad \int dv = \int e^{x}$

$dx = \cos x \, du \qquad v = e^{x}$

$\int u \, dx = uv - \int v \, du$

$\int u \, dx = \sin x \times e^{x} - \int e^{x} \cos x \, dx$

$= e^{x} \sin x - \int e^{x} \cos x \, dx$

$\int e^{x} \cos x \, dx$

$u = \cos x \qquad dv = e^{x}$

$\frac{du}{dx} = -\sin x \qquad \int dv = \int e^{x}$

$du = -\sin x \, dx \qquad v = e^{x}$

$\int u \, dv = uv - \int v \, du$

$= \cos x \times e^{x} - \int e^{x} - \sin x \, dx$

$= e^{x} \cos x - \int -e^{x} \sin x \, dx$

$= e^{x} \cos x + \int e^{x} \sin x \, dx$

$\int e^x \sin x\,dx = e^x \sin x - (e^x \cos x + \int e^x \sin x\,dx)$

$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$

add $\int e^x \sin x\,dx$ to both sides

$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x$

$2\int e^x \sin x\,dx = e^x(\sin x - \cos x)$

$\int e^x \sin x\,dx = \frac{e^x}{2}(\sin x - \cos x) + C$

(2) $\int 2x^2 \ln x\,dx$

$u = \ln x \qquad dv = 2x^2$

$\frac{du}{dx} = \frac{1}{x} \qquad \int dv = \int 2x^2$

$du = \frac{1}{x}dx \qquad v = \frac{2x^3}{3}$

$\int u\,dv = uv - \int v\,du$

$= \ln x \cdot \frac{2x^3}{3} - \int \frac{2x^3}{3}\cdot\frac{1}{x}dx$

$= \frac{2x^3}{3}\ln x - \int \frac{2x^3}{3}dx$

$= \frac{2x^3}{3}\ln x - \frac{2x^3}{9} + C$

$2\left(\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right) + C$

(3) $\int x^2 \sin x \, dx$

$u = x^2 \qquad dv = \sin x$

$\frac{du}{dx} = 2x \qquad \int dv = \int \sin x$

$du = 2x \, dx \qquad v = -\cos x$

$\int u \, dv = uv - \int v \, du$

$= x^2(-\cos x) - \int -\cos x \times 2x \, dx$

$= -x^2 \cos x - \int -2x \cos x \, dx$

$= -x^2 \cos x + \int 2x \cos x \, dx$

$\int 2x \cos x \, dx$

$u = 2x \qquad dv = \cos x$

$\frac{du}{dx} = 2 \qquad \int dv = \int \cos x$

$du = 2 \, dx \qquad v = \sin x$

$\int u \, dx = uv - \int v \, du$

$= 2x \sin x - \int \sin x \cdot 2 \, dx$

$= 2x \sin x - \int 2 \sin x \, dx$

$= 2x \sin x - 2 \int \sin x \, dx$

$= 2x \sin x - (2 - \cos x)$

$= 2x \sin x + 2 \cos x$

$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$

$= 2x\sin x + 2\cos x - x^2\cos x + C$

$= 2x\sin x + \cos x(2 - x^2) + C$

(4) $\int x\cos x\,dx$

$u = x \qquad dv = \cos x$

$\frac{du}{dx} = 1 \qquad \int dv = \int \cos x$

$du = dx \qquad v = \sin x$

$\int u\,dv = uv - \int v\,du$

$= x\sin x - \int \sin x\,dx$

$= x\sin x - (-\cos x) + C$

$= x\sin x + \cos x + C$