Find the integral of the following:

1. $\int e^x \sin x \, dx$

$u = \sin x \qquad dv = e^x$

$du = \cos x \, dx \qquad v = e^x$

$\int u \, dv = uv - \int v \, du$

$\sin x (e^x) - \int e^x \cos x \, dx$

$e^x \sin x - \int e^x \cos x \, dx$

$\int u = \cos x \qquad dv = e^x$

$du = -\sin x \, dx \qquad v = e^x$

$\int \cos x (e^x) - \int e^x (-\sin x) dx$

$\int e^x \cos x + \int e^x \sin x \, dx$

$e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

Let $I = \int e^x \sin x \, dx$

$I = e^x \sin x - e^x \cos x - I$

$2I = e^x \sin x - e^x \cos x$

$I = \dfrac{e^x \sin x - e^x \cos x}{2}$

$\therefore \int e^x \sin x \, dx = \frac{1}{2} [e^x \sin x - e^x \cos x] + C$

2. $\int 2x^2 \ln x \, dx$

$u = \ln x \qquad dv = 2x^2$

$du = \frac{1}{x} dx \qquad v = \frac{2}{3} x^3$

$\ln x \cdot \dfrac{2x^3}{3} - \dfrac{2}{3} \int x^3 \cdot \dfrac{1}{x}$

$\ln x \cdot \dfrac{2x^3}{3} - \dfrac{2}{3} \int x^2$

$\left(\ln x \cdot \dfrac{2x^3}{3} \right) - \dfrac{2x^3}{9} + C$

$\therefore 2x^2 \ln x \, dx = \left(\ln x \cdot \dfrac{2x^3}{3} \right) - \dfrac{2x^3}{9} + C$

3. $\int x^2 \sin x \, dx$

$u = x^2 \qquad dv = \sin x$

$du = 2x \qquad v = -\cos x$

$\dfrac{du}{dx} = 2x$

$\int u \, dv = uv - \int v \, du$

$x^2 (-\cos x) - \int -\cos x \, (2x \, dx)$

$= -x^2 \cos x - \int -2x \cos x \, dx$

Let $u = -2x$, $du = \cos x$

$du/dx = -2x \qquad v = \sin x$

$\therefore (-2x)(\sin x) = \int (\sin x)(-2) \, dx$

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$= 2x\sin x - (-2)\int \sin x \, dx$

$-2x\sin x - (-2) - \cos x + C$

$-2x\sin x - 2\cos x + C$

$\therefore \int x^2 \sin x \, dx =$

$-x^2\cos x - 2x\sin x - 2\cos x + C$

4. $x\cos x \, dx$

$\int x \cos x \, dx$

$u = x \qquad dv = \cos x$

$\frac{du}{dx} = 1 \qquad v = \sin x$

$\int u \, dv = uv - \int v \, du$

$= x(\sin x) - \int \sin x \, dx$

$\therefore \int x\cos x \, dx = x\sin x + \cos x + C$